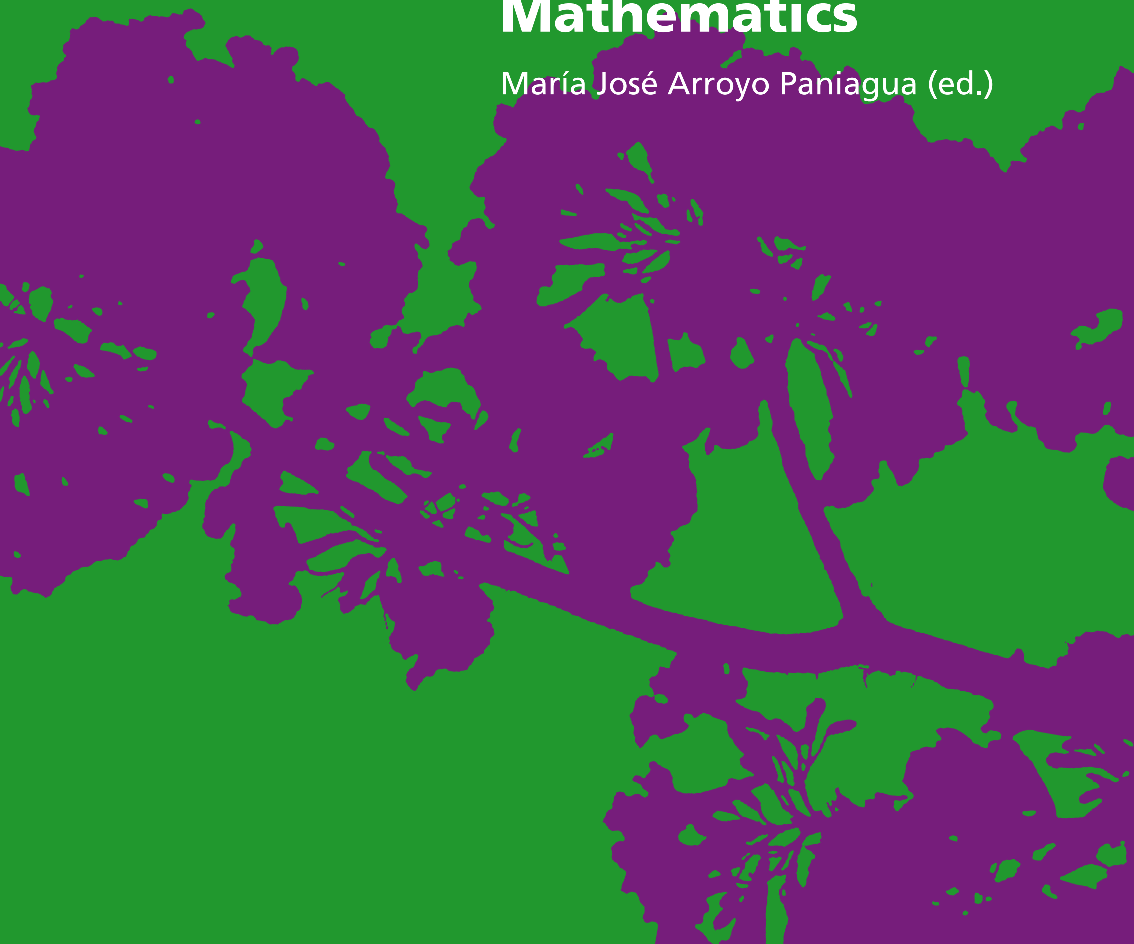
Tuning

Latin America

Higher Education in Latin America: reflections and perspectives on **Mathematics**

María José Arroyo Paniagua (ed.)



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Tuning Latin America Project

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Index

Tuning: past, present and future
An introduction

Major changes have taken place worldwide in higher education over the last 10 years, although this has been a period of intense reflection particularly for Latin America, insofar as the strengthening of existing bonds between nations has been promoted and the region has started to be considered as being increasingly close. These last 10 years also represent the transition time between Tuning starting out as an initiative that arose as a response to European needs and going on to become a worldwide proposal. Tuning Latin America marks the start of the Tuning internationalisation process. The concern with thinking how to progress towards a shared area for universities while respecting traditions and diversity ceased to be an exclusive concern for Europeans and has become a global need.

It is important to provide the reader of this work with some definitions of Tuning. Firstly, we can say that Tuning is **a network of learning communities**. Tuning may be understood as being a network of inter-connected academic and student communities that reflects on issues, engages in debate, designs instruments and compares results. They are experts that have been brought together around a discipline within a spirit of mutual trust. They work in international and intercultural groups and are totally respectful of independence on an institutional, national and regional level, exchanging knowledge and experiences. They develop a common language to problems in higher education to be understood and take part in designing a set of tools that are useful for their work, and which have been devised and produced by other academics. They are able to take part in a platform for reflection and action about higher education - a platform made up of hundreds of communities from different countries. They are responsible for developing reference points for disciplines that represent a system for designing top quality qualifications which are shared by many. They are open to the possibility of creating networks with many regions of the world within their own field and feel that they are responsible for this task.

Tuning is built on each person that forms part of that community and shares ideas, initiatives and doubts. It is global because it has pursued an approach based on worldwide standards while at the same time remaining both local and regional, respecting the specific features and demands of each context. The recent publication: *Communities of Learning: Networks and the Shaping of Intellectual Identity in Europe, 1100-1500* (Crossley Encanto, 2011) takes all the new ideas into consideration which are developed within a community context, whether of an academic, social or religious nature or simply as a network of friends. The challenge facing Tuning communities is to gain an impact on the development of higher education in its regions. Secondly, Tuning is a **methodology** with well-designed steps and a dynamic outlook that enables different contexts to be adapted. The methodology has a clear aim: to build qualifications which are compatible, comparable, are relevant to society and with top levels of both quality and excellence, while preserving the valuable diversity deriving from the traditions of each country involved. These requirements demand a collaborative methodology based on consensus which is developed by experts from different fields who are representatives of their disciplines, and who have the ability to understand local, national and regional situations.

This methodology has been developed around **three core themes**: the first is the **qualification profile**, the second is the **syllabus** and the third refers to the **trajectories of those who learn**.

The qualification profile enjoys a key position in Tuning. After a lengthy period of reflection and debate within Tuning projects in different regions (Latin America, Africa, Russia), the qualifications profile may be defined as being a combination of forces revolving around four core points:

- The region's needs (from local issues to the international context).
- The meta-profile of the area.

- The taking into consideration of future trends in the profession and society.

- The specific mission of the university.

The question of **social relevance** is essential for the design of profiles. Without doubt, any analysis of the relationship existing between university and society lies at the heart of the matter of relevance in higher education. Tuning's aim is to identify and meet the needs of the production sector, the economy, society as a whole and the needs of each student within a particular area of study – measured by specific social and cultural contexts. With a view to achieving a balance between these different needs, goals and aspirations, Tuning has consulted leading people, key local thinkers and experts from industry, both learned and civil society and working parties that include all those interested. An initial period of this phase of the methodology is linked to general competences. Each thematic area involves the preparation of a list of general competences deemed relevant from the standpoint of the region concerned. This task ends when the group has widely discussed and reached consensus about a selection of specific competences, and the task is also performed with specific competences. Once the means of consultation has been agreed and the process completed, the final stage in this practical exercise involving the search for social relevance refers to an analysis of results. This is done jointly by the group, and special care is taken not to lose any contributions from the different cultural perceptions that might illustrate understanding of the specific reality.

Once lists of the general and specific agreed, consulted and analysed competences had been obtained, a new phase got underway over these last two years that is related to the **development of meta-profiles for the area** under consideration. For Tuning methodology, meta-profiles represent the structures of the areas and combinations of competences (general and specific) that lend identity to the disciplinary area concerned. Meta-profiles are mental constructions that categorise competences in recognisable components and illustrate their inter-relations.

Furthermore, thinking about education means becoming involved in the present, while above all also looking towards the future – thinking about social needs, and anticipating political, economic and cultural changes. This means also taking into account and trying to foresee the challenges that those future professionals will have to face and the impact that certain profiles of qualifications is likely to have, as designing profiles is basically an exercise that involves looking to the future. Within the present context, designing degree courses takes time in order for them to be planned and developed and their approval obtained. Students need years to achieve results and mature in terms of their learning. Then, once they have finished their degree, they will need to serve, be prepared to act, innovate and transform future societies in which they will find new challenges. Qualification profiles will in turn need to look more to the future than the present. For this reason, it is important to take an element into consideration that should always be taken into account, which are future trends both in terms of the specific field and society in general. This is a sign of quality in design. Tuning Latin America embarked on a methodology so as to incorporate an **analysis of future trends into the design of profiles**. The first step therefore involved the search for a methodology to devise future scenarios following an analysis of the most relevant studies in education by focusing on the changing role of higher educational establishments and trends in educational policies. A methodology was chosen based on in-depth interviews with a dual focus: on the one hand, there were questions that led to the construction of future scenarios on a general society level, their changes and impact. This part needed to serve as a basis for the second part, which dealt specifically with the features of the area in itself, their transformation in general terms in addition to any possible changes in the degree courses themselves that might have tended to disappear, re-emerge or be transformed. The final part sought to anticipate the possible impact on competences based on present coordinates and the driving forces behind change.

There is a final element that has to be taken into account when constructing the profiles, which is linked to the **relationship with the university where the qualification is taught**. The mark and mission of the university must be reflected in the profile of the qualification that is being designed.

The second core theme of the methodology is linked to **syllabuses**, and this is where two very important Tuning components come into play: on the one hand, students' work volume, which has been reflected in an agreement to establish the Latin American Reference Credit (CLAR), and all studies are based on this and, on the other, the intense reflection process into how to learn, teach and assess competences. Both aspects have been covered in Tuning Latin America.

Lastly, an important area is opened up for future reflection about the **trajectories of those who learn** – a system that proposes focusing on the student leads one to consider how to position oneself from that standpoint so as to be able to interpret and improve the reality in which we find ourselves.

Finally, Tuning is a **project** and as such came into existence with a set of objectives and results and within a particular context. It arose from the needs of the Europe of 1999, and as a result of the challenge laid down by the 1999 Bologna Declaration. Since 2003, Tuning has become a project that goes beyond European borders, in so doing embarking on intense work in Latin America. Two very specific problems faced by the university as a global entity were pinpointed: on the one hand, the need to modernise, reformulate and make syllabuses more flexible in the light of new trends, society's requirements and changing results in a vertiginous world and, on the other, which is linked closely to the first problem, the importance of transcending limits imposed by staff in terms of learning, by providing training that would enable what has been learnt to be recognised beyond institutional local, national and regional borders. The Tuning Latin America project thus emerged which, in its first phase (2004-2007), sought to engage in a debate whose goal was to identify and exchange information and improve collaboration between higher educational establishments, with a view to developing the quality, effectiveness and transparency of qualifications and syllabuses.

This new phase of **Tuning Latin America (2011-2013)** started life on already-fertile terrain – the fruits of the previous phase and in view of the current demand on the part of Latin American universities and governments to facilitate the continuation of the process that had already been embarked on. The aim of the new Tuning phase in the region was to help build a Higher Education Area in Latin America. This challenge takes the form of four very specific central working themes: a deeper understanding of agreements involving **designing meta-profiles and profiles in the 15 thematic areas** included in the project (Administration, Agronomy, Architecture, Law, Education, Nursing, Physics, Geology, History, Information Technology, Civil Engineering, Mathematics, Medicine, Psychology and Chemistry); contributing to **reflections on future scenarios for new professions**; promoting the joint construction of **methodological strategies in order to develop and assess the training of competences**; and designing **a system of academic reference credits (CLAR-Latin American Reference Credit)** to facilitate recognition of studies in Latin America as a region that can be articulated with systems from other regions.

The Tuning door to the world was Latin America, although this internationalisation of the process wouldn't have gone far if it hadn't been for a group of prestigious academics (230 representatives of Latin American universities), who not only believed in the project, but also used their time and creativity to make it possible from north to south and west to east across the extensive, diverse continent that is Latin America. This was a group of experts in different thematic areas that would go on to study in depth and gain weight in terms of their scope and educational force, and in their commitment to a joint task that history had placed in their hands. Their ideas, experiences and determination paved the way and enabled the results which are embodied in this publication to be achieved.

Yet the Tuning Latin America project was also designed, coordinated and administered by Latin Americans from the region itself, via the committed work carried out by Maida Marty Maleta, Margarethe Macke and Paulina Sierra. This also established a type of *modus operandi*, conduct, appropriation of the idea and of deep respect for how this was going to take shape in the region. When other regions decided to join Tuning, there would henceforth be a local team that would be responsible for considering what to emphasize - specific features, the new elements that would need to be created to meet needs which, even though many of them might have common characteristics within a globalised world, involve dimensions specific to the region, are worthy of major respect and are, in many cases, of major scope and importance.

There is another pillar on this path which should be mentioned: the coordinators of the thematic areas (César Esquetini Cáceres-Coordinator of the Area of Administration; Jovita Antonieta Miranda Barrios-Coordinator of the Area of Agronomy; Samuel Ricardo Vélez González-Coordinator of the Area of Architecture; Loussia Musse Felix-Coordinator of the Area of Law; Ana María Montaño López-Coordinator of the Area of Education; Luz Angélica Muñoz González-Coordinator of the Area of Nursing; Armando Fernández Guillermet-Coordinator of the Area of Physics; Iván Soto-Coordinator of the

Area of Geology; Darío Campos Rodríguez-Coordinator of the Area of History; José Lino Contreras Véliz-Coordinator of the Area of Information Technology; Alba Maritza Guerrero Spínola-Coordinator of the Area of Civil Engineering; María José Arroyo Paniagua-Coordinator of the Area of Mathematics; Christel Hanne-Coordinator of the Area of Medicine; Diego Efrén Rodríguez Cárdenas-Coordinator of the Area of Psychology; and Gustavo Pedraza Aboytes-Coordinator of the Area of Chemistry). These academics, chosen according to the thematic groups to which they belonged, were the driving forces behind the building of bridges and strengthening of links between the project's Management Committee of which they formed a part and their thematic groups which they always held in high regard, respected and felt proud to represent. Likewise, they enabled there to be valuable articulation between the different areas, showing great ability to admire and listen to the specific elements attached to each discipline in order to incorporate, take on board, learn and develop each contribution – the bridges between the dream and the reality. Because they had to carve new paths in many cases to make the ideas possible, design new approaches in the actual language of the area and the considerations proposed, and to ensure that the group would think about them from the standpoint of the specific nature of each discipline. Following group construction, the process always requires a solid framework based on generosity and rigour. In this respect, the coordinators were able to ensure that the project would achieve specific successful results.

Apart from the contribution made by the 15 thematic areas, Tuning Latin America has also been accompanied by a further two transversal groups: the Social Innovation group (coordinated by Aurelio Villa) and the 18 National Tuning Centres. The former created new dimensions that enabled debates to be enriched and an area for future reflection on thematic areas to be opened up. Without doubt, this new area of work will give rise to innovative perspectives to enable those involved to continue thinking about top quality higher education that is connected to the social needs of any given context.

The second transversal group about which one should recognise the major role played comprises the National Tuning Centres – an area of representatives from the highest authorities of university policies from each of the 18 countries in the region. These centres accompanied the project right from the outset, supported and opened up the reality of their national contexts to the needs or possibilities developed by Tuning, understood them, engaged in dialogue with others, disseminated them and constituted reference points when seeking genuine anchors and possible goals. The National Centres have been a contribution from Latin America to the Tuning project, insofar as they have contextualised debates by assuming and adapting the results to local times and needs.

We find ourselves coming to the end of a phase of intense work. The results envisaged over the course of the project have succeeded all expectations. The fruits of this effort and commitment take the form of the reflections on the area of Mathematics that will be provided below. This process comes to an end in view of the challenge faced in continuing to make our educational structures more dynamic, encouraging mobility and meeting points within Latin America, while at the same time building the bridges required with other regions on the planet.

This is the challenge facing Tuning in Latin America.

July 2013

Pablo Beneitone, Julia González and *Robert Wagenaar*

To the reader

The results of the work carried out by those who form part of the Subject Area of Mathematics within the Tuning Latin America: Educational and Social Innovation Project are provided in this booklet. It is structured in six sections: the first, titled «Brief description of the Area of Mathematics», gives a summarised overview of the situation regarding mathematics in the region in order to highlight the context within which the work was carried out. The second, «Meta-profile for the Area of Mathematics» provides the graduate meta-profile designed for the mathematics profession, describing the way in which it was constructed, while at the same time contrasting the level of attainment and development of competences with the graduate profiles already established in programmes and curricula or in terms of their implementation in participant universities. The third, «Future scenarios for the Area of Mathematics and the profession», offers a standpoint put forward by professionals from the world of mathematics who practice their profession in different sectors of society (through a set of interviews). The fourth, «Observations concerning student workload», shows the results of an exercise by means of which observations on student workload were noted from the standpoint of both lecturers and students from participant universities. This exercise involved applying a questionnaire in educational establishments and processing the information obtained. The fifth, «Teaching, learning and assessment strategies for competences», describes those obtained via the analyses conducted and experiences gained from teachers' associations at participant universities. Two examples are provided with regard to the generic competence *Capacity for oral and written communication* and the specific competence *Ability to express oneself correctly using the language of mathematics*: the planning of activities on a course that is common to all pre-graduate training in mathematics regarding ordinary differential equations, taking into consideration the competences to be developed within it, and a virtual learning environment for a differential calculus and introductory integral course. Both examples reflect the ways in which the attainment of the competences established in the student profile within the programme can be implemented and back-up for lecturers provided with this aim in mind. Lastly, the general conclusions reached by those involved in the area are provided once the work within the project was completed. A directory of all participants in the Area of Mathematics is also included.

1

Brief description of the Area of Mathematics

It was already mentioned in the introduction that the Tuning Latin America: Educational and Social Innovation Project dates back to when the project was set in motion during the years 2004 to 2007. A map of the area of mathematics was provided in the Final Report (Beneitone et al 2007).

Six years have passed since then, and it is natural to assume that many aspects of the area be maintained without any major variation. However, some changes have been noted. A brief summary is provided below in order to offer the context within which the activities pursued in the area were carried out within this project and that in which mathematics professionals are trained nowadays.

The different degree programmes associated with mathematics in most of the countries are referred to as *licenciaturas* (honours degree programmes), except for in Colombia and Brazil. In Colombia, mathematics programmes are referred to as *carreras* (degree courses). The term *licenciatura* is used to refer to studies that involve teacher training. There are two options for qualifications in Brazil - *bachillerato* (LIT. baccalaureate) and *licenciatura* - and both levels of professionals are referred to as mathematicians, *Licenciado* in Mathematics in Brazil means being qualified as a teacher in basic education, whereas a *Bachiller* in Mathematics refers to a future researcher who will go on to work in universities and research centres.

The names of the degree programmes have been retained and remain varied, such as: *Licenciatura* in Science (Mathematics), *Licenciatura* in Mathematics, *Licenciatura* in Mathematics, *Licenciatura* in Applied Mathematics, *Bachillerato* in Mathematics and *Bachillerato* in Applied Mathematics, *Ingenieria* (Engineering) in Mathematics, *Licenciatura* in Physics-Mathematics, *Licenciatura* in Mathematical Science, *Licenciatura* in Statistics, *Licenciatura* in Statistical Science, *Licenciatura* in Actuarial Science and Actuary.

The duration of studies in higher educational establishments ranges between four and six years. The requirement of preparing a thesis or dissertation in order to obtain this first university degree is not uniform – Cuba is the only country in the region with a unified syllabus.

Degree programmes leading to qualifications that focus on teacher training are referred to by equally differing names, such as *Licenciatura* in Mathematical Education, *Licenciatura* in Teaching of Mathematics, *Licenciatura* in Educational Mathematics, *Profesorado* (Teacher Training) in Middle School Education specialising in Mathematics, *Profesorado* in Middle and Higher Education in Mathematics, *Profesorado* in Middle School Education specialising in Mathematics and Computer Studies, *Profesorado* in Middle School Education specialising in Mathematics and Physical Sciences, *Licenciatura* in Educational Science with mention of Mathematics, *Licenciatura* in Educational Science with mention of Mathematics and Physics, *Licenciatura* in Educational Science with mention of Information Technology and Mathematics, and *Profesorado* specialising in Mathematics.

As in the case of the project that was set in motion over the period 2004-2007, this time the Subject Area of Mathematics decided to focus their activities on programmes that train mathematicians to enable them to perform in different sectors of society, not only in a career within higher educational establishments and research centres.

Current educational policies in many countries favour internationalisation and collaboration. This has given rise to and shall continue to give rise to an increase in joint efforts to promote programmes and projects of common interest, among which is included mobility on the part of academics and students.

The Central American University Council (CSUCA) is working hard to promote the education of mathematicians and physicists in this region at both under-graduate and post-graduate level.

Accreditation agencies or bodies which are responsible for recognising the quality of undergraduate and post-graduate programmes have changed some of their criteria and indicators, and educational establishments plan actions that enable them to establish improvements in their curricula.

Mathematics programmes which are included in the training of professionals in all fields are not the exclusive responsibility of mathematicians, but rather, they share this work with engineers or physicists. The departmental system is present in universities in some countries (such as Mexico, Brazil and Cuba), although this is not a general feature.

Major concern persists about the quality of mathematics learning at all levels in the region. Generally speaking, the position of research in mathematics and its impact on mathematics teaching and mathematics teachers differs. The establishment and growth of post-graduate studies in mathematics still faces major challenges for the development of research in Latin America – in countries where this does exist and is well-established, it may nonetheless still not extend to the whole country.

An attempt has been made to strengthen post-graduate programmes in recent years, and these have grown in terms of what they are geared towards. Profiles linked to applied mathematics are being increasingly incorporated in degree programmes. As is natural, this is closely linked to the evolution and growth of the task of lecturers and researchers in academic departments within educational establishments, as well as their experiences of links to other bodies. These circumstances have not radically changed in six years, although the natural changeover of academic staff, policies aimed at retaining talent and internationalisation-oriented activities have fostered the diversification of projects in accordance with the needs of modern society.

In nearly all countries, universities, research centres and different scientific associations have resolutely promoted the dissemination of knowledge and the exchange of mathematical tasks both within the country concerned and beyond its borders. Periodic meetings and national and international conferences are organised within different specialist branches of mathematics for such purposes.

Most universities are autonomous and themselves define the curricula for *licenciaturas* or *carreras*, and for MA and PhD programmes. They are regulated by their own university legislation and are voluntarily subject to self-assessment, external assessment and accreditation processes - authorities that may be decentralised from ministries in charge of education. Although universities are autonomous, programmes in some of them may be subject to approval and assessment by ministries in charge of higher education.

On the other hand, the need is being increasingly recognised in educational establishments to adapt and improve teaching and learning processes in programmes in order to help to develop in students the competences deemed desirable for a graduate in the profession. There is the conviction that this helps to achieve integral training in accordance with the needs of future professionals. In most countries, changes have been proposed to curricula and syllabuses in order to promote the development of those competences that integrate skills so as to deal with problems within the current context and to encourage students to put forward solutions, improve on them and use them in new situations.

The professional practice of mathematicians continues to become diversified, as professional niches are being opened up in different social sectors. Fields of work outside the academic sector are to be found mainly in service industries – in insurance and finance companies, in banking and in economic management, in public health statistics, in national census offices, in the modelling of industrial and engineering processes and in scientific research centres of different types. To a lesser extent, they are also found in governmental sectors that focus on public health, economic planning and the control of energy resources.

2

Meta-profile for the Area of Mathematics

This section is structured in four parts: a description of the meta-profile itself and the methodology pursued to construct it; a diagram/outline representing it; a comparison of the meta-profile with graduate profiles in the region and with the prevailing situation in curricula and syllabuses to do with vocational training in mathematics; and the degree of relevance of the competences involved in the meta-profile.

The curricular convergence required for the recognition of capacities in mathematics professionals in the region makes it necessary to emphasise the characteristics required of graduates. It is desirable for such characteristics to be developed in higher educational establishments which, in turn, will contribute towards better development of their future professional activities, irrespective of the field of work in which the graduate may be involved – whether university lecturer-researcher, post-graduate student or providing professional consultancy services in different areas of production.

2.1. Graduate meta-profile for the Mathematics degree programme

The graduate on a Mathematics degree programme is an ethical professional who shows human sensitivity, social responsibility and citizenship, and is willing to learn, keep constantly up-to-date and deal with new problems in different spheres of activity.

They will have undertaken solid training in terms of their own knowledge, abilities and skills in their area of study and their profession, gained a command of basic concepts in higher mathematics and develop logical arguments by clearly identifying hypotheses and conclusions.

They are characterised by their great capacity for abstraction and analysis, including the logical development of mathematical theories and relationships between them, and are qualified to embark on mathematical research under expert guidance.

They permanently apply knowledge in practice and identify and confront problems, formulating them in mathematical language in such a way as to facilitate their analysis and solution. They help construct mathematical models from real situations and by analysing experimental data.

They possess basic knowledge of the teaching-learning process in mathematics, formulate and administer projects and use information and communications technologies.

They express themselves correctly and effectively both orally and in writing, have a command of the language of mathematics and use mathematical reasoning and draw relevant conclusions with clarity, accuracy and in a suitable manner for the target audience.

They read, write and explain documents in English in order to interact with the international academic community in their area of knowledge.

They communicate with other professionals who do not work in the field of mathematics and provide advice in the application of mathematics in their respective areas of work.

They possess the interpersonal skills required to interact with other people within different contexts and to work in multidisciplinary teams.

The work carried out by the Area of Mathematics during the first phase of the Tuning Latin America project was used as a basis for determining the meta-profile. In their final report for 2007 (ibid.) 27 generic competences were established as being desirable for development by all graduates enrolled in higher education, together with 23 specific competences for the Mathematics.

To construct the meta-profile, all the generic and specific competences determined in the first phase of the project were analysed, and a decision was made to establish four dimensions in order to group and link them together. These four dimensions were considered to be equally important in the education of the future professional, as each one has a specific purpose, they are not dealt with as isolated parts, and they are linked and articulated with different levels of emphasis within the curricular network, in accordance with the levels of development established in the programme according to accepted institutional definitions.

Without establishing hierarchies between them, the dimension related to the attitude shown by a future professional was initially given prominence as a key element in their education and future professional practice, followed by the dimension related to know-how within the discipline and the underlying tools required for this. The final two dimensions were those related to communication and interpersonal skills of the future professional in mathematics. Generic competences (GC) and specific competences (SC) appear in the table in accordance with the numbering used in the report from the first phase of the project. The description of each dimensions and the association of generic and specific competences within each of them was as follows:

Attitudinal Dimension: refers to the competences required by a mathematician who shows human sensitivity and a commitment to society.

GC 5 Social responsibility and citizenship. GC 10 Ability to learn and keep constantly up-to-date. GC 12 Critical and self-critical capacity. GC 16 Capacity for decision-making. GC 19 Capacity to motivate towards common goals. GC 20 Commitment to conservation of the environment. GC 21 Commitment to their socio-cultural milieu. GC 22 Appreciation and respect for diversity and multiculturalism. GC 24 Ability to work independently. GC 26 Ethical commitment. GC 27 Commitment to quality.	SC 14 Willingness to deal with new problems in different areas.

Disciplinary Dimension (cognitive and procedural): refers to those competences that a mathematician needs to develop solidly in terms of the knowledge, abilities and skills required of the field and their applications.

GC 1 Capacity for abstraction, analysis and synthesis. GC 2 Ability to apply knowledge in practice. GC 3 Ability to organise and plan time. GC 4 Knowledge about the area of studies and profession. GC 8 Skills in the use of information and communications technologies. GC 9 Research capacity.	SC 1 Command of basic concepts in higher mathematics. SC 2 Ability to construct and develop logical arguments by clearly identifying hypotheses and conclusions. SC 4 Capacity for abstraction, including the logical development of mathematical theories and relationships between them. SC 5 Ability to formulate problems in mathematical language in such a way as to facilitate their analysis and solution.

GC 11 Skills in searching for, processing and analysing information from different sources. GC 14 Creative capacity. GC 15 Ability to identify, consider and deal with problems. GC 25 Ability to formulate and administer projects.	SC 6 Knowledge of the historical evolution of the fundamental concepts of mathematics. SC 7 Ability to embark on mathematical research under expert guidance. SC 8 Ability to formulate optimisation problems, take decisions and interpret solutions within the original contexts of the problems themselves. SC 9 Ability to help construct mathematical models from real situations. SC 10 Ability to use computer numeric and symbolic calculus tools in order to consider and deal with problems. SC 11 Skill in quantitative reasoning. SC 12 Ability to understand problems and extract the essential elements from them. SC 13 Ability to extract qualitative information from quantitative data. CE 15 Ability to work with experimental data and contribute towards their analysis. CE 19 Basic knowledge of the teaching-learning process in mathematics. CE 20 Command of elementary mathematics, i.e. what needs to be included in pre-university education. CE 21 Ability to take part in preparing mathematical training programmes on a pre-university level. CE 22 Ability to detect inconsistencies.

Communicational Dimension: refers to those competences that qualify the mathematician to express themselves correctly and effectively both orally and in writing.

GC 6 Capacity for oral and written communication. GC 7 Capacity for communication in a second language.	CE 3 Ability to express oneself correctly using the language of mathematics. CE 16 Ability to communicate with other professionals who are not mathematicians and provide them with advice in the application of mathematics in their respective areas of work. CE 18 Ability to use mathematical reasoning and draw relevant conclusions with clarity, accuracy and in a suitable manner for the target audience, both orally and in writing. CE 23 Knowledge of English for the purpose of reading, writing and describing documents, as well as the communicating with other specialists.

Relational Dimension: refers to those competences required to ensure that the mathematician is capable of interacting with other people within different contexts.

GC 13 Ability to act in new situations. GC 17 Capacity for teamwork. GC 18 Interpersonal skills. GC 23 Ability to work within international contexts.	SC 17 Ability to work in interdisciplinary teams.

When analysing the list of generic and specific competences attached to the area that were defined in the first phase of the project, it was clear that some of them form a part of other, more broad-ranging ones – which are included or at least necessarily implied. Their importance is acknowledged and remained explicitly stated as such in terms of their distribution within the different dimensions. Nonetheless, a reduction in the number of competences grouped together in each dimension might help one to understand the definition of the meta-profile better and, where appropriate, its implementation. A selected list of both generic and specific competences was determined with this division. It should be pointed out that observations by members of the area about the importance of the competences selected coincided with their assessment provided in the previously-mentioned 2007 report.

However, it should be noted that competences not selected in this document maintain their value by themselves, although they may be developed as a result of the development of other ones and that their evolution may refer to training background and to the interest on the part of the future professional – and would be associated with the specific academic profile attached to each educational establishment.

The selected generic and specific competences that gave rise to the meta-profile in each dimension are:

Attitudinal Dimension: refers to the competences required to train a mathematician who shows human sensitivity and a commitment to society.

GC 5 Social responsibility and citizenship. GC 10 Ability to learn and keep constantly up-to-date. GC 26 Ethical commitment.	SC 14 Willingness to deal with new problems in different areas.

Disciplinary Dimension (cognitive and procedural): refers to those competences that a mathematician needs to develop solidly in terms of the knowledge, abilities and skills required of the field and their applications.

GC 1 Capacity for abstraction, analysis and synthesis. GC 2 Ability to apply knowledge in practice. GC 4 Knowledge about the area of studies and profession. GC 8 Skills in the use of information and communications technologies. GC 15 Ability to identify, consider and deal with problems. GC 25 Ability to formulate and administer projects.	SC 1 Command of basic concepts in higher mathematics. SC 2 Ability to construct and develop logical arguments by clearly identifying hypotheses and conclusions. SC 4 Capacity for abstraction, including the logical development of mathematical theories and relationships between them. SC 5 Ability to formulate problems in mathematical language in such a way as to facilitate their analysis and solution. SC 7 Ability to embark on mathematical research under expert guidance. SC 9 Ability to help construct mathematical models from real situations. SC 15 Ability to work with experimental data and contribute towards their analysis. SC 19 Basic knowledge of the teaching-learning process in mathematics.

Communicational Dimension: refers to those competences that qualify the mathematician to enable them to express themselves correctly and effectively both orally and in writing.

GC 6 Capacity for oral and written communication. GC 7 Capacity for communication in a second language.	SC 3 Ability to express oneself correctly using the language of mathematics. SC 16 Ability to communicate with other professionals who are not mathematicians and provide them with advice in the application of mathematics in their respective areas of work. SC 18 Ability to use mathematical reasoning and draw relevant conclusions with clarity, accuracy and in a suitable manner for the target audience, both orally and in writing. SC 23 Knowledge of English for the purpose of reading, writing and describing documents, as well as communicating with other specialists.

Relational Dimension: refers to those competences required to ensure that the mathematician be capable of interacting with other people within different contexts.

GC 13 Ability to act in new situations. GC 17 Capacity for teamwork. GC 18 Interpersonal skills. GC 23 Ability to work within international contexts.	SC 17 Ability to work in interdisciplinary teams.

The undergraduate meta-profile for the profession of mathematician takes into consideration the professional problems that graduates will need to deal with once completing their studies.

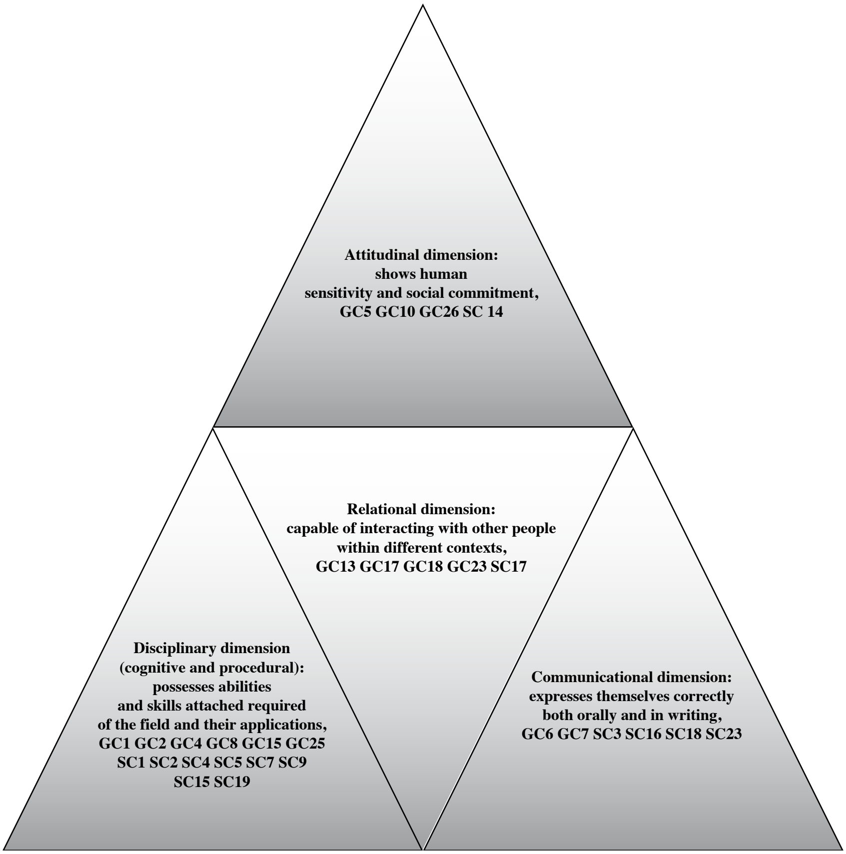
This constitutes the basis for continuing with post-graduate education and MA and PhD studies, although at the same time should be sufficient to resolve a series of professional problems common to different spheres of activity that may facilitate entry into the job market. For this reason, the meta-profile is closely linked to the minimum disciplinary content resulting from an analysis of different curricula and programmes which, generally speaking, academics participating in this study think inform the vocational education of mathematicians offered by universities in the region.

Below is listed the minimum content worked on by the Area of Mathematics during the first phase of the project, listing the subject matter that should be covered in any undergraduate mathematics programme:

	Minimum content
Elementary Geometry	Congruence of figures, areas of flat figures, resemblance of figures, regular polygons.
Analytical Geometry	Elementary geometry of plane and space, coordinate and conical systems.
Differential Geometry	Curves and surfaces.
Linear Algebra	Linear and matrix equation systems, vector systems and linear applications, own values and vectors
Abstract Algebra	Sets, ratios and applications, elementary algebraic structures: Z, Z_n, Q, R, C, polynomials, groups, subgroups, normal groups, rings, subrings and ideal ones.
Number Theory	Euclidean algorithm, Fermat's little theorem, Euler's theorem, Lagrange's theorem, fundamental theorem of arithmetic.
Calculus	Numeric successions and series, continuity, differentiation and integration of functions of one or more real variables, line and surface integrals and classical theorems of calculus.
Differential Equations	First-order differential equations, superior-order differential linear equations, differential linear equation systems, introduction to the qualitative analysis of equations and applications.
Complex Variable	Analytical functions, complex integration, Liouvilles's theorem, maximum modulus theorem, the principle of argument, Rouché's theorem, singularities and residues.
Mathematical Analysis	R^n, topology, continuity and differentiability of real functions and of real variables, Riemann integral, successions and series of functions, inverse function theorem, implicit function theorem.
Measurement and Integration and Functional Analysis	Fundamental results of the theory of measurement and integration, functional analysis and the operator theory.

	Minimum content
Topology	Basic concepts of topology, continuity, homeomorphisms, compactness, connexity and separation.
Discrete Mathematics	Combinatory, analysis of algorithms and graph theory.
Numeric Methods	Study of errors, floating point arithmetic, methods for solving equations and linear and non-linear equation systems, interpolation polynomials, numeric interpolation, numeric differentiation and integration, least-squares method, approximation functions, numeric solving of ordinary differential equations.
Optimisation	Basic optimisation methods, in particular linear programming, non-linear programming, whole programming and graph theory.
Probability and Statistics	Random variables, probability space distribution and density functions, sampling, statistical inference, linear models and some aspects of multivariate and stochastic process analysis.
Programming and Algorithms	Development of skills for the construction of algorithms via the use of calculation equipment, operative systems, data base systems, in a visual object-oriented programming language.
Logic and Basic Principles	Formal languages and systems, calculation of subjects and predicates, computability and decidability.
History and Methodology of Mathematics	Overview of the historical development of mathematics and its fundamental philosophical problems.
Mathematical Modelling	Integrating course devoted to solving certain problems.
Didactics of Mathematics	In accordance with the features offered at the educational establishment or subjects covered during mobility stays.
Physics	Classical mechanics and the continuous medium.
Chemistry or Biology	In accordance with the specialist subjects offered at the educational establishment.
Social Science and Humanities	In accordance with the specialist subjects offered at the educational establishment.

2.2. Outline of the meta-profile



2.3. Regional comparison of the meta-profile with mathematics programmes and curricula

Once conclusions had been reached about the meta-profile for the Area of Mathematics for the region, this was then compared with existing programmes at the Latin American universities taking part in the project. To do so, it was deemed relevant to carry out a documentary analysis of the content of the programmes and curricula. In the case of Colombia and Cuba, all programmes existing in the country were taken into account, while in Mexico, four educational establishments were selected. The programme corresponding to the university taking part in the project was analysed in the case of the other countries.

Mathematics degree programmes in the region contain the description of professional or graduate profiles. There were very few that provide an explicitly-designed profile in terms of competences – Colombia has two and Cuba three. However, most of the competences that form part of the meta-profile designed by the Area of Mathematics within the Tuning Latin America Project are included in documentation, either explicitly or implicitly. Moreover, changes have been made to programmes at several educational establishments in order to specify the development of the competences desired in them.

Declarations are made in graduate profiles about social responsibility, citizenship and ethical commitment, more as values that need to be cultivated in educational establishments and their communities than as aspects that need to be developed in students. The willingness to learn, keep constantly up-to-date and deal with new problems are what is most commonly found.

The declaration and purpose of developing competences related to the disciplinary dimension is present in some form or another in all profiles, with priority being given to those competences attached to a mathematician's tasks. Generic competences related to the use of ICTs and the formulation and administering of projects are not evident among the main objectives that are suggested to be developed in profiles.

For their part, generic competences included in the communicational and relational dimensions are virtually absent in most profiles, whereas they tend to favour some specific competences within these dimensions, such as the ability to express oneself correctly using the language of mathematics, the ability to communicate with professionals who are not mathematicians and the ability to work in multidisciplinary teams.

2.4. Degree of relevance of the competences involved in the meta-profile

To be able to establish an approximation regarding the degree of relevance of the competences involved in the meta-profile, surveys were conducted and/or consultations made with some lecturers and those in charge (leaders) of groups of subjects or disciplines —schools, faculties— at universities who are responsible for developing mathematics curricula at universities, in order for their responses to lend support to the conclusions drawn from the diagnosis.

The survey was structured by taking into consideration the selected competences contained in the dimensions that gave rise to the meta-profile. Each competence was assigned an indicator value in accordance with the following scale:

- 5 - Activities carried out within the discipline —subject, course— contribute fully towards training in this competence.
- 4 - Activities carried out within the discipline —subject, course— contribute towards training in this competence, but some aspects still need to be included in order to achieve full development.
- 3 - Activities carried out within the discipline —subject, course— contribute partially towards training in this competence, but nonetheless guarantee it to an acceptable extent.
- 2 - Activities carried out within the discipline —subject, course— are insufficient for the purpose of training in this competence.
- 1 - This competence is not included among the objectives set out in the discipline – subject, course.

The sample of lecturers and academic leaders who took part in the survey amounted to 86 lecturers in total.

It should be pointed out that when conducting the survey, Cuba deemed it necessary to establish six indicators and also asked about all and every one of the competences originally taken into consideration. In the case of Colombia, a scale was not used, but rather, an assessment was made about the development of the competences selected in the meta-profile on several courses and within the general programme.

By taking into account the fact that a real process is being judged in order to offer an overview of the region, a qualitative assessment is provided below of the results of the surveys applied. The conclusions drawn are as follows:

The development of the ability to learn and keep constantly up-to-date and to deal with new problems is greatly stressed in mathematics programmes; activities are carried out in all programmes within the different disciplines that contribute either fully or partially to developing this competence.

As for social responsibility and citizenship, their development is considered to be undertaken transversally across the curriculum and there is no homogenous drive to do so in evidence in all participant educational establishments. Ethical principles are cultivated in the daily tasks performed in communities, with an emphasis being placed on work carried out with other people. It is deemed necessary to encourage these capacities more, as activities geared towards developing or strengthening them in several educational establishments are insufficient or even non-existent.

Most competences such as knowledge, abilities and skills attached to mathematics, a command of its basic concepts, the construction and development of logical arguments by clearly identifying hypotheses and conclusions, and the capacity for abstraction, analysis and synthesis attain a high level of development during the programmes and are worked on at different levels or structures within the curriculum.

It can be noted that those competences related to the application of knowledge in practice need to be developed further, as at present emphasis in programmes is placed on the formal and abstract rather than on the applied. Although the time set aside for solving real problems is improving, there still exist gaps that need to be filled; with regard to the optimum use of information and communication technologies, the use of and access to digital libraries is growing, although the use and exploitation of available software or packages for different specialist subjects in daily interaction between lecturers and students is not growing at the same speed.

As far as knowledge of teaching-learning processes is concerned, the degree of relevance is low and it is hoped that this will improve in the future, as in the case with project management – a competence that would appear not to be well-developed in most programmes.

The degree of relevance of communication-oriented competences is high in terms of communication towards mathematics and professionals working in this field as, except for one university, activities are carried out in all educational establishments that are geared towards the proper development of this competence. However, this does not occur in the case of communication towards other professional spheres of activity. As for command of a second language, this competence is only measured as a degree requirement in most programmes, and it is though that this should be developed gradually on their different levels or structures.

Lastly, the capacities established in the relational dimension have not been suitably developed – individual work is regarded and promoted far more than interdisciplinary teamwork and within an international context. Activities and even subjects have been designed in many educational establishments in order to reinforce the development of these capacities.

3

Future scenarios for the Area of Mathematics and the profession

Members of the Subject Area of Mathematics conducted 24 interviews in order to establish views about future scenarios in twenty years' time. Prior to this, each member of the area was requested to consult at their respective educational establishments which people to interview, selecting those with broad view of the professional field and a distinguished career.

Each interview was recorded and summarised in accordance with the following questions:

1. Looking to the future, the mathematician's contribution, collaboration and knowledge will be required over the next twenty years. What might be possible scenarios? In which professional fields?
2. What professional problems will emerge and require the application of mathematical models or methods? Consider which other professions the mathematician will need to interact with.
3. Are there at present any problems of this type that have not been taken into consideration?
4. Which models, techniques and procedures are expected to be included in vocational training in mathematics to ensure that these problems are tackled? Which competences will be required?
5. What is going to the mathematician's role in the future?

Below is provided a summary of the main conclusions drawn.

a) *Brief description of interviewees*

Interviewees have wide experience and have different professional profiles: some work in the area of science and academia while others work in research centres that deal with problems from other areas of knowledge, in industry and in governmental organisations. In some cases, they have over thirty years' experience, hold Master's degrees and PhDs, and their work covers a wide range of disciplines in mathematics and its applications in others spheres of science and development, as well as education.

b) *Characterisation of future scenarios taken into consideration*

Extremely varied scenarios can be envisaged in which diverse specialists carry out studies of different real problems on a macro level and in which mathematical models and methods are used to explain phenomena.

In addition to branches that are already well-established in Physics and Engineering, major applications will also be developed in Mathematics, in processes which are studied in Ecology, Biology, Biotechnology, Chemistry, Neuroscience, Nanotechnology, Probability, Communications Technology and Microelectronics, Meteorology, Medicine and the energy industry, to name just some. The same will happen in social sciences in view of their growing mathematisation, mainly in Economics and Finance, Sociology, Psychology and Pedagogy. Advances made in science will appear with greater intensity and faster, and will continue to give rise to breakthroughs that will pave the way for new fields to be studied.

As its presence can be assured in any field, mathematics offers the professional in this discipline the chance to form part of such teams and help, via their creative intervention, to solve the complex problems that will need to be dealt with.

c) *Professions that can be envisaged in each scenario*

No distinct professions are envisaged in new scenarios. Mathematics will become consolidated as a degree programme that can and must be maintained in the future with solid theoretical foundations within a flexible structure that is able to adapt to new situations and enable competences emerging as a result of society's development to be incorporated. Specialist areas that may emerge in modelling and research within mathematics itself and its applications will become the focus of attention via optional pre-graduate subjects or during the post-graduate training stage.

More than new professions, reference will be made to new professional problems, the solution to which will require the cooperation of mathematicians. They will consequently be increasingly in demand.

High regard will in turn be placed within the mathematics profession itself on work carried out in what is referred to as Applied Mathematics. This contrasts with what is at present known as Abstract and Pure Mathematics, as the mathematician's professional activity will increasingly require interaction with a range of professionals – among them, physicists, doctors, biologists, economists and engineers.

d) *Competences that will be required of these professions*

The competences established in the graduate meta-profile for the Mathematics degree programme will remain valid.

Basically, a new *mentality* will be required of new professionals in the field of mathematics, who will increasingly come to form part of multidisciplinary teams with specialists from other sciences and experts in other techniques, in which they will provide their knowledge and experience more than actually *becoming specialists in other sciences*. This new mentality should be based on solid mathematical foundations and complemented by access to other fields of knowledge via the curriculum and in which they will be given intellectual preparation that will enable permanent independent learning to be developed. This will provide them with a broad overview of the changing world in which they will be carrying out their professional activity.

e) *Other relevant comments about the future*

The world of work is undergoing constant change, and innovation and economic development will need to have a bearing on social development, which entails countless implications for education and professions. Certain activities will disappear while others will emerge, many of them deriving from the technological development in which the demand for qualified professionals will increase. One is advised of the need to address these aspects in universities.

To deal with future problems, the mathematician will need to be educated so as to avoid current problems existing in their teaching and learning from a level of basic training to vocational training. To this end, mathematics teachers will need to be professionals that have undergone solid preparation in the subject and are able to provide an effective mathematical contribution towards solving these problems.

It is also mentioned that the fundamental role that will need to be played by the mathematician in the future will be work to establish links between the different branches of mathematics and show the need to use mathematical models and methods with effective solutions in order to deal with a range of complex yet interesting problems in science and technology. This raises a challenge for mathematicians in the future - one that involves not only dealing with current problems of interest both to the mathematician's own development and to the development of society. A harmonious balance is required in the education of the mathematician between basic mathematics and applied mathematics: Mathematics and its Applications.

One problem that is also envisaged is that of mathematics teaching at all levels, bearing in mind that this is a problem in which mathematicians will need to take an active part.

Mathematicians in the future will need to participate in public life in society, with corresponding recognition of the community.

4

Observations concerning student workload

An activity was carried out in the course of the project to estimate the real workload assumed by students in order to study the subjects corresponding to a given period (quarters, four-month periods, semesters or school year) from the standpoint of planning and from the viewpoint of both lecturers and students.

Each area defined a period to be consulted in order to proceed with the relevant consultations. The Area of Mathematics considered the school period ending in 2011 as corresponding to the second year of the programme and thus adhering to the scheduling for all activities required for this purpose, which covered the period from the beginning of the month of January to 30th March 2012. The corresponding programmes were listed in each university. These were then reported to the project coordinators, and the senior and assistant lecturers giving the courses and all students who had enrolled and passed each subject were located in order to enable responses to be given to the questionnaires. At least ten students per subject were consulted, with an average four subjects being taken into consideration over the period chosen. Bearing in mind that the questionnaire was applied per subject, in many cases, a student had to respond to questionnaires about more than one subject – one for each subject they had passed over that period. The interviews were scheduled and the results loaded into a centralised system attached to the project. Interviewees were provided with a brief explanation about the project and the purpose of the questionnaire. It should be stated that face-to-face activities are deemed to be those in which both lecturer and student take part.

The questionnaire that was applied to lecturers contained the following questions:

1. Name of the thematic area ___.
2. Name of the university ___.
3. Name of the degree programme ___.
4. Name of the subject ___.
5. Duration in weeks of the academic period in accordance with the syllabus ___.
6. How many minutes are there in a teaching hour in your subject? ___ minutes ___don't know ___ no answer.
7. How many hours of face-to-face teaching activities did your subject comprise? ___hours ___don't know ___ no answer.
8. How many weeks of real face-to-face teaching activities did your subject comprise, including assessments? ___weeks ___don't know ___ no answer.
9. How many hours of face-to-face teaching activities did your subject comprise per week? ___hours ___don't know ___ no answer.
10. How many total hours do you think students set aside during the academic period in order to pass their subject, taking into account all face-to-face and non-face-to-face activities? ___hours ___don't know ___no answer.
11. Of the following non-face-to-face activities, indicate which ones you used to encourage independent student work. Indicate the number of clock hours you think students needed to do so.

    a) Reading of texts or bibliography ___yes ___no ___hours ___ don't know ___ no answer.

    b) Preparation and development of work ___yes ___no __hours ___don't know ___no answer.

c) Field work ___yes ___no ___hours ___don't know ___ no answer.

d) Laboratory ___yes ___no ___hours ___don't know ___ no answer.

e) Preparation and development of written work ___yes ___no ___hours ___don't know ___ no answer.

f) Virtual activities ___yes ___no ___hours ___don't know ___ no answer.

g) Study for assessment purposes ___yes ___no ___hours ___ don't know ___ no answer.

h), i) and j) Other: specify: ___ ___yes ___no ___hours ___don't know ___ no answer.

12. How many hours a week on average do you think students set aside for face-to-face and non-face-to-face activities in the subject? ___yes ___no ___hours ___ don't know ___ no answer.

13. When planning your subject, did you take into consideration the number of non-face-to-face hours that students required in order to carry out the activities? ___yes ___no ___hours ___don't know _____no answer.

14. Did you compare this estimate of non-face-to-face hours with that of students? ___yes ___no ___hours ___don't know ___ no answer.

The questionnaire that was applied to students was as follows:

1. Name of the thematic area ___.

2. Name of the university ___.

3. Name of the degree programme ___.

4. Name of the subject ___.

5. Duration in weeks of the academic period in accordance with the syllabus ___.

6. How many minutes are there in a teaching hour in the subject? ___ minutes ___don't know ___ no answer.

7. How many hours of face-to-face teaching activities did the subject comprise? ___hours ___don't know ___ no answer.

8. How many weeks of real face-to-face teaching activities did the subject comprise, including assessments? ___weeks ___don't know ___ no answer.

9. How many hours of face-to-face teaching activities did the subject comprise per week? ___hours ___don't know ___ no answer.

10. How many total hours do you think you set aside during the academic period in order to pass your subject, taking into account all face-to-face and non-face-to-face activities? ___hours ___don't know ___no answer.

11. Of the following non-face-to-face activities, indicate which ones you carried out while studying for this subject. Indicate the number of clock hours you think you needed to do so:

    a) Reading of texts or bibliography ___yes ___no ___hours ___ don't know ___ no answer.

    b) Preparation and development of work ___yes ___no __hours ___don't know ___no answer.

    c) Field work ___yes ___no ___hours ___don't know ___ no answer.

    d) Laboratory ___yes ___no ___hours ___don't know ___ no answer.

    e) Preparation and development of written work ___yes ___no ___hours ___don't know ___ no answer.

    f) Virtual activities ___yes ___no ___hours ___don't know ___ no answer.

g) Study for assessment purposes ___yes ___no ___hours ___ don't know ___ no answer.

h) i) and j) Other: specify: ___ ___yes ___no ___hours ___don't know ___ no answer.

12. How many hours a week on average do you think you set aside for face-to-face and non-face-to-face activities in the subject? ___yes ___no ___hours ___ don't know ___ no answer.

13. Did you plan the number of non-face-to-face hours that you required in order to carry out the activities? ___yes ___no ___hours ___ don't know ____no answer.

14. Did the lecturer contrast this estimate of non-face-to-face hours with you? ___yes ___no ___hours ___don't know ___ no answer.

In terms of the results obtained from this activity in the project, the number of respondents was 10,086; 189 educational establishments took part with their respective teaching periods (quarters, four-month periods, semesters and school year). The bases for the analyses and values provided were determined by the project coordinators.

It was noted that the average number of weeks per academic period varies between 16 and 18 weeks, i.e. 32 to 39 weeks of activities during the year, with most establishments working according to semesters.

The mean number of hours that it was estimated were required by students to carry out all the face-to-face and non-face-to-face activities taken into consideration by all lecturers and students from all thematic areas included in the project was 1,546 hours.

In all degree programmes covered by the project it was noted that lecturers estimate that a student set aside a mean 636.88 hours in order to pass a subject during one academic period, with all activities being taken into consideration that need to be carried out either face-to-face or non-face-to-face, whereas the mean value noted by students was 610.8 hours. The means corresponding to degree programmes in Mathematics during one academic period by lecturers was 525.5 hours, whereas for students it was 753.59 hours.

On average, it was noted that in all degree programmes covered by the project, lecturers estimate that their students set aside 46.94 hours a week, whereas students estimate 52.36 hours. In the case of Mathematics degree programmes, lecturers estimate that students set aside 51.1 hours a week, whereas students estimate 56.49 hours.

In terms of non-face-to-face activities that lecturers use to encourage independent work by students on Mathematics degree programmes, over 90% of lecturers mention *Reading texts*, followed by *Study for assessment purposes* with over 80%, *Written work* was used by over 60% of lecturers, *Virtual activities* accounted for nearly 40%, and a little less than 20% of lecturers mention using *Laboratory work* and *Field work*. For their part, 90% of students refer to *Reading texts*, nearly 90% *Study for assessment purposes*, 20% refer to *Virtual activities*, over 10% carried out *Laboratory work* and less than 5% claim they carried out *Field work*.

In Mathematics, 60% of lecturers answered that they planned their subject by taking into account the number of hours students should set aside for non-face-to-face activities, whereas only 35% of students stated that they planned the non-face-to-face hours they needed to set aside in order to carry out their activities.

Lastly, approximately 24% of mathematics lecturers stated that they contrasted their estimate of number of hours required for their subject with students, while for their part, 17% of students stated that lecturers contrasted their estimate of non-face-to-face hours required for the subject with the former.

It is important to stress the fact of taking time into consideration that students carry out their activities in order to achieve learning objectives (number of hours of independent work), and that this is as important as face-to-face activities.

In this respect, it is important to encourage reflection about face-to-face and non-face-to-face hours, which is required throughout the teaching-learning process. The activity described in this section gave rise to reflection on the part of lecturers and students in assuming responsibility for trying to ensure better distribution of the tasks to be performed and the importance of planning and organising this in subjects.

5

Teaching, learning and assessment strategies for the competences selected

The Area of Mathematics defined the meta-profile of the mathematician for the Latin American region as a professional who, among other competences, «*expresses themselves correctly and effectively both orally and in writing, has a command of the language of mathematics and uses mathematical reasoning and draws relevant conclusions with clarity, accuracy and in a suitable manner for the target audience*».

Consequently, the group deemed it relevant to study the teaching, learning and assessment strategies for these competences in programmes educating future professionals in mathematics.

- Generic competence: capacity for oral and written communication.
- Specific competence: ability to express oneself correctly using the language of mathematics.

The competences selected carry a great deal of weight and are essential for mathematics education.

Since the outset, attention has been drawn to the fact that teaching, learning and assessment strategies are correlated with forms of organisation, structure and academic and pedagogic models, as well as individuals' experience and culture that help to implement them.

To put together this report, prior work was carried out in educational establishments in which both documents and reflections with groups of academics in each of them were compiled.

It was observed that even though programme structures vary immensely among the different participant establishments, these are established according to organisational and functional models; the structure is broken down into quarters, semesters or years; and programmes are referred to by a number of names and are sub-divided into disciplinary cores or disciplines. One aspect in common was that levels of achievement are expected in each of the competences selected, in addition to their location within the curricular network or within this specifically in a subject. This may constitute a response, on the one hand, to a general situation regarding the undergraduate profile in the region in terms of the generic competence chosen and, on the other, to a natural development of the specific competence selected within the syllabus.

In all establishments, the teaching and learning process forms part of a system in which development of competences – and, consequently, achievement in the graduate profile – is obtained via a coinciding of all their competences in the time it takes to implement the programme. It is acknowledged that the different components that make up the curriculum of the programme help to a differing extent to develop competences, and it is important for development of the objectives established in its structure and in courses to be explicitly expressed.

Different ways of implementing the programme exist. Most subjects are taught via face-to-face lectures, practical classes and seminars in the classroom, as well as lecturer-student interaction. The latter is done via «meetings», in which the lecturer's work involves providing guidance about the work students need to carry out individually or independently as part of a team, while at the same time controlling and checking the results obtained from the most recent meeting held. Practical laboratory work is also carried out, and if linked to mathematics subjects, this is deemed to be practical work that uses specific software as a tool.

Another type of activity that is becoming increasingly prominent is the use of virtual environments by way of support for communication between lecturers and students, and especially between students.

In terms of the generic competence *Capacity for oral and written communication*, any university graduate should to be able to give explanations orally, and in writing, including producing and written texts that may include texts, images and sound.

Although no subject in a programme may be devoted to this purpose, work must be carried out to develop this competence during the course of the students' period of study. Learning and dealing with spelling and editing may not be explicit objectives within a subject in particular, but these are checked in assessment activities, although this may take place with greater or lesser intensity. A lecturer often places more emphasis on the subject knowledge demonstrated, but at the same time recognises that the medium for obtaining the result expected requires the capacity for oral and written communication.

With regard to the specific competence *Capacity to express oneself correctly using the language of mathematics*, it is impossible to think of a mathematics professional who does not have this.

As for the relationship existing between the curriculum and the development of competences in general, below is provided a brief —albeit not exhaustive— description of some structures for which some universities in countries represented in the Mathematics group have opted for.

In Bolivia, a programme is structured in three stages: The Basic Stage, from the first to the fourth semesters, the Formative Stage, from the fifth to the eighth, and the Professional Stage in the ninth and tenth. The following subjects appear in the first stage: Calculus I and II, Mathematical Statistics, Basic Physics I and II, Algebra, Linear Algebra, Technical English, Geometry, Information Technology, Operative Research I and II, Vector Analysis, Numeric Analysis I, Didactics of Mathematics and Ordinary Differential Equations. In the Formative Stage are subjects such as Analysis I, II and III, Abstract Algebra I and II, Topology I and II, Research Methodology, Optimisation Methods, Functional Analysis and some optional subjects. In the Professional Stage are found subjects such as Equations in Partial Derivatives, Numeric Analysis and Research Workshop I and II.

In Cuba, the latest version of the degree programme from the academic year 2007-2008 contains a basic compulsory core for the whole country. This includes both generic and specific competences that the graduate should possess in essential knowledge and skills about the profession. This basic core is organised in disciplines that cover general education (Social Sciences, Physical Education, English Language and Preparation for Defence) and basic mathematical disciplines (Programming and Algorithms, Mathematical Analysis, Algebra, Geometry and Topology, Numeric Mathematics, Probability and Statistics, Optimisation, Differential Equations, Theory of Complex Variable Functions, Measurement and Integration and Functional Analysis, and History and Methodology of Mathematics). In Cuba, «discipline» is understood to mean a didactic unit that may consist of one or more subjects referring to content belonging to the same branch of knowledge. This basic core is complemented by its own curriculum, which comprises compulsory, optional and specific elective subjects at each university which are structured in an integrating disciplinary core from the first to the last year, under the name «Professional Practice for the Mathematician». The BA honours degree is of 4 years' duration and concludes with a State Examination, although the most outstanding students may conclude by defending diploma work (BA honours thesis).

In Colombia, the programme is structured according to subjects over eight semesters. Such subjects as the following are taught in the first year: Introduction to Calculus and Calculus I, Euclidean Geometry and Introduction to Computers. In the second year Calculus II, Linear Algebra, Set Theory, Epistemology and History of Science, Ordinary Differential Equations, Physics I and Algebra I are taught; in the third, Estimation and Interference, Analysis I and II and Algebra II and III and in the fourth such subjects as Numeric Analysis, Topology, Complex Variable Functions, Introduction to Research, Professional Ethics and General Culture Seminar are found.

In Chile, the programme is structured in two stages of two years each, divided into semesters: the Basic stage and the Specialist stage. The former is structured according to courses such as Algebra and Geometry I and II, Experimental Methods, Calculus I and II, Mechanics I and II, Linear Algebra I and Differential Equations. The Specialist stage includes, among other subjects, those such as Groups and Rings, Real Analysis, Abstract Analysis, Probability and Statistics, Complex Variable, Research Unit and Scientific English.

In Mexico, the programme on which the analysis was carried out is divided into formative stages and subjects are distributed across the four years of duration, with each year comprising three terms. There is a complementary course pre-programme module, and enrolling on this depends on the student's entry qualification. Basic courses include Introduction to Mathematical Thought, Mechanics and Fluids and Differential and Integral Calculus. Vocational courses include subjects such as Discrete Mathematics, two courses devoted to Linear Algebra and to Differential Equations, others to Advanced Calculus and Mathematical Analysis, Mathematical Models, Probability and Statistics and Complex Variable by way of compulsory courses. The Complementary Stage includes Research Seminars and optional subjects from Social Sciences or Biological and Health Sciences, together with others devoted to studying mathematical disciplines in depth. The amount of time set aside may vary according to the different subjects at the different stages.

In most countries, specialization for the mathematicians is complemented in Master's programmes and PhDs in Mathematical Science, which extend for a further six-year period (two for the Master's degree and four for the PhD).

In Brazil and Guatemala, programmes devoted to teacher education in Mathematics from two participant universities are described in non-exhaustive terms by the educational establishment to which members of the Area of Mathematics belong. It was noted that the strategies, teaching and assessment attached to competences in the case of the competences mentioned coincided with those of the programmes in pure mathematics. In these establishments, the analysis was carried out according to specific subjects and the results reported.

In Brazil, the programme is structured in 8 semesters. In the first year are included subjects such as Educational Reality of Brazil, Portuguese Language, Geometric Design, Function Topics, Philosophy of Education, Trigonometry, Differential and Integral Calculus I and Physics I; in the second year are found subjects such as Psychology of Learning and Differential and Integral Calculus II; in the third year appear Evolution Topics of Mathematical Thought, Differential Calculus IV, Mathematical Logic, Financial Mathematics, Linear Algebra, and Basic Principles of Teaching and Learning Processes in Mathematics; and in the fourth are included subjects such as Complex Numbers and Polynomial Equations, Algebra I, Differential Equations, Algebra II, Introduction to Numeric Methods and Introduction to Real Analysis.

In Guatemala, the programme is structured in 12 semesters. The first year covers Ethics, Physics I, Calculus I, Statistics I, Language, Sociological Study of Guatemala and Conceptual Physics I; in the second year includes subjects such as Pre-Calculus II, Statistics, Psychology of Learning, Calculus I, Didactic Programming and Theory of Knowledge; in the third year, among other subjects, are Physics II, Calculus II, Statistics II and Philosophy of Education, and History; in the fourth are Calculus III, Design, Development and Curricular Assessment, Electromagnetism, History of Mathematics and Physics, Introduction to Differential Equations and Quantitative Research Methods in Education I; in the fifth, among others, are the subjects Problem-Solving Strategies, Mathematicians and Scientists, Quantitative Methods in Education and Linear Algebra; and the sixth mainly focuses on Virtual Environment Strategies for Learning Mathematics and Physics and thesis work.

5.1. Teaching, learning and assessment strategies for the generic competence: capacity for oral and written communication

5.1.1. *Description*

This refers to a command of written, oral and visual expression that manifests itself through work – summaries, reviews, essays, presentations and projects – based on the reading and intelligent handling of relevant sources. The work taken into account in these terms expresses personal standpoints based on reasoned and reasonable arguments. This capacity involves explaining a theme or problem of a professional nature, formulating its scope, describing the relevant facts to enable it to be understood and establishing the logical steps leading to a convincing conclusion, whereby proper understanding of the issues explained and their relevance are shown.

Level of development

- Low:
  - Expresses their ideas both orally and in writing in a legible and comprehensible manner with a logical structure taking into consideration the context within which the communication takes place.

— Refers to indispensable elements and provides some references for argument.

— Identifies some suitable elements and makes correct use of spelling, punctuation, grammar and vocabulary.

— Shows consistency and clarity.

- Average:

— Addresses an audience in order to inform them about a given content, puts forward ideas with reasoned argument, adapts the structure of the discourse to the audience's objectives and demands, and incorporates elements that facilitate understanding and interest on the part of the listener or reader.

— Their work fulfils its task in terms of dealing with the topic, includes an outline of personal suggestions and standpoints and relevant references that support the argument.

— Makes relevant use of language; spelling, punctuation, grammar and vocabulary are appropriate within the context of the discipline.

— Avoids repetition by using synonyms and phrases and deploys clear language.

- High:

— Expresses work they have prepared themselves both orally and in writing that is both in keeping with reality and innovative, with a tendency to transform reality; shows order in terms of their ideas, and clarity in terms of expression and incorporation of those elements deemed necessary to persuade their audience.

— Reveals knowledge of the topic, uses an original standpoint and puts forward their arguments supported by sufficient relevant references.

— Makes appropriate use of language; spelling, punctuation, grammar and vocabulary are most suitable within the context of the discipline; language is clear and varied.

— Expresses their arguments convincingly and clearly, using concise and fluent language.

5.1.2. *Learning results*

Level of development

- Low:

  - Identifies and understands the vocabulary in order to interpret scientific literature and text books corresponding to their level of studies.
  - Reads analytically and interprets problems and different types of text.
  - Coherently expresses their ideas about bibliographic consultation made about topics related to courses via brief papers.
  - Presents documents —summaries of texts— in which they highlight the correct use of spelling, grammar and punctuation.
  - Adapts their oral and written communication to the context within which ideas are expressed or exchanged.

- Average:

  - Reads a range of texts analytically and extracts information from them.
  - Uses and develops strategies that may help them to produce written texts with coherence and clarity, in which they make correct use of grammatical rules about a range of reading material, particularly in mathematics.
  - Understands, express and analyses written texts and oral exposés.
  - Communicates and conveys the knowledge acquired via oral exposés in which a coherent structure is established.
  - Takes part in forums and events in a broader academic environment than the classroom in which they use the means necessary to arouse interest from the audience and increase understanding of ideas.
  - Provides explanations using data and basic principles, backed up by images, graphs and time lines.

- High:

  — Uses a wide range of relevant vocabulary when expressing themselves orally and in writing.

  — Prepares, drafts and produces complex written texts such as essays and monographs with suitable use of the Spanish language.

  — Prepares, drafts and produces academic texts such as research reports and degree theses in which they use elements deemed appropriate for written communication (justification, theoretical framework, methodology, bibliographic references, etc.), within the academic field in general and mathematics in particular, and also uses mathematical word processors.

  — Gives oral exposés about their research work or pre-graduate thesis in which they provide reasoning and conclusions with clarity, accuracy and in a suitable way for the target audience; makes use in these of tools that enable them to convey knowledge to other professionals who are not mathematicians.

  — Embodies an analysis of the information in their texts, essays and presentations, the same applying to discussion, conclusions and recommendations found in the course of the research work – all in a logical order, fluently, coherently and suitably presented.

  — Offers a defence of their work both orally and in writing before different types of public.

  — Expresses, formulates and interprets the concepts and results of their mathematical work with scientific rigour.

5.1.3. *Teaching and learning strategies*

- Encourage reading of literature in general.

- Propose reading and then comment on it in the group, thus favouring students' reading comprehension.

- Via bibliographic research and explanations from the lecturer, propose the drafting of summaries in which students use words and systematise ideas regarding comprehension of the concepts that appear in the material to be consulted, and express the meanings and possibilities for applying the content that is being studied.

- Formulate questions from a story, account or essay that enable the student to study a text in depth and distinguish the essential parts of it.

- Students review and compare their own work with that submitted by their classmates.

- Contrast the results observed with students via their oral presentations and written work.

- Provide scripts to lend support to students in preparing their work.

- Set written and oral examinations and hand over results to students so that they may analyse their mistakes and identify correct arguments and procedures.

- Have printed and electronic material at their disposal for use by students that provide access to a wide range of high quality examples.

- Use virtual learning environments for group discussions about tasks undertaken in face-to-face environments or as a complement to the latter, by means of which students and lecturers may observe, question, argue, interpret and deduce matters, thus encouraging critical reflection and the conceiving of new ideas.

- Propose problems and challenges in virtual environments that need to be resolved via group discussions in which all students may propose the steps to be taken and provide explanations about the resources to be used. Interaction is fed by discussions among those who take part —both lecturer and student— who are required to make an effort to understand and make themselves understood in writing, this forming the basis for communication in such media.

- Encourage use of blogs, the presentation of papers in different academic forums such as seminars, debates, public speaking competitions, drafting of progress reports on projects to be developed on the different courses, and the design of informative pamphlets or news bulletins using different media.

- Encourage explicit recognition of the importance of developing this capacity among lecturers in order for them to incorporate it effectively in their course exercises.

- Include the activities to be carried out in syllabuses and course planning by the lecturer in which the specific products that lend support to the development and assessment of the capacity for oral and written communication in students are identified.

5.1.4. *Strategies for assessment of results*

Assessment is in itself subject to study, research, reflection and research action on the part of lecturers who are committed to developing knowledge of better quality at different levels of education. Research results point to the importance of involving the student in such a way that ensures they are responsible for their own learning process. This process is, in turn, related to the intellectual independence of the student, who identifies their potential and also any difficulties they are facing, and who is able to plan their acquisition of skills within a continuous learning process.

Assessment under these conditions becomes formative process, in which both lecturers and students are committed to modifying the ways they work as new knowledge becomes consolidated and new indicators are created that guide the direction taken by the learning process. This approach is a change from the traditional summative approach.

Assessment as a formative process requires the mapping out and understanding of how learning takes place, and what difficulties, obstacles and progress need to be improved. Conceived thus, assessment provides information to enable the lecturer to schedule pedagogic interventions by putting forward problems and challenges to help students establish relations and develop valuable skills and conduct. Assessment is therefore developed within a continuous process of diagnosis and as a source of re-invention in terms of teaching practice.

- Assessment strategies identified:

  — Preparing tables in order to assess learning results in terms of language, content, structure and organisation, argument, conclusions, bibliography used and presentation by lecturers collectively.

— Ensuring that lecturers apply collectively made agreements in order to assess results in students' written work, tasks, examinations, essays and oral and visual presentations, etc.

— Establishing guidelines for assessment of individual work or teamwork carried out by students such as: written examinations, public speaking, papers, forums or debates, blogs, essays, progress reports, participation in seminars, exposés about a project, articles, approaches to researching theses, the introduction, the theoretical framework, the methodology and bibliographic references of the thesis, and the oral and written presentation of the draft thesis and its final presentation.

5.2. Teaching, learning and assessment strategies for the specific competence: ability to express oneself correctly using the language of mathematics

5.2.1. *Description*

This competence involves the use of terms and symbols deemed appropriate for the field or area of mathematics in which they are being used. Ideas that emerge from reflective processes involving abstraction, analysis and synthesis are embodied through mathematical language. Their proper use in turn involves the presentation of a succession of reasoned arguments which are formulated, organised and discussed and from which conclusions are drawn in an orderly and logical manner. Language is deployed when applying logic in the deductive method for appropriation and/or generation of mathematical knowledge. This is the tool that enables problems that emerge from mathematics itself and from other disciplines to be formulated. It includes the ability to explain a topic or problem of a mathematical nature, formulate its scope, describe the relevant facts to ensure it is understood and establish the logical steps that lead to a convincing and truthful conclusion.

Level of development

- Low:

— Reads about outlines of problems analytically and seeks solutions by using variables and taking equations into consideration.

— Handles wording and expressions which contain symbols and formulas.

— Uses mathematical symbols correctly.

— Expresses their reasoned arguments in an orderly and logical manner in elementary mathematical models, whether orally or in writing.

- Average:

— Uses the language of formal logic correctly.

— Presents mathematical concepts using the wording of the mathematical discourse with clear organisation and structure of the mathematical objects they use; expresses the distinction made and relationship existing between diverse mathematical statements such as definitions, theorems, examples, conditioned assertions and tests, etc., both orally and in writing.

— Expresses their description and interpretation of symbolic and formal language both orally and in writing, in addition to their relationship with natural language.

— Unlike the first level, this is closely linked to the contextualisation of mathematics via the approach and solution to problems that may be dealt with using mathematical tools.

- High:

— Distinguishes between and discerns from among different ways of representing mathematical objects, depending on the situation and purpose.

— Prepares complex texts (research reports, pre-graduate work, theses) in accordance with publication standards in mathematics, in which reasoned arguments and their conclusions are shown clearly and accurately and in an appropriate manner for the target audience.

— Gives oral exposés using the language of mathematics correctly, in order to share knowledge with their peers.

— Explains the analysis, interpretation and development of their own models and strategies both orally and in writing, including tests and general considerations.

— Explains what they recognise and gather from mathematics in a range of real situations or with real problems, in order to use them to find a solution.

5.2.2. *Learning results*

In accordance with their level of studies, the student should be able to:

- Express the approach, analysis and development of mathematical models both orally and in writing.
- Read and explain demonstrations in diverse areas of mathematics and also put together their own demonstrations.
- Use the syntax of abstract and symbolic language accurately in mathematical structures, including the use of information technology (IT) tools in some cases.
- Correctly translate problems into the language of mathematics that have been drafted in non-technical language, and vice-versa.
- Explain the main points of an informative article about mathematics.
- Read books about mathematics on an introductory level in all fields of this discipline, as they progress in the syllabus.
- Express and formulate a wide range of problems.
- Communicate with professionals both from the world of mathematics and from other disciplines.

5.2.3. *Teaching and learning strategies*

- Initiate activity during the teaching period by requesting a summary of the activity carried out during previous periods.
- Select texts about the topics to be covered.
- Approach problems that may have different solutions and discuss them.

- Encourage students to make comparisons between examples and to establish any differences and relations that may exist between them.
- Approach general considerations about resolved situations.
- Approach bibliographic searches both as individuals and in teams.
- Develop activities in which students may put forward their arguments both orally and in writing, whether in the form of class notes, tasks, reports, examinations or exposés in face-to-face activities (classes and seminars). Of these, ensure that suitable IT tools are used, where appropriate, in which special care needs to be taken regarding the correct use of grammar and syntax attached to mathematical language.
- Establish oral or written dialogues via virtual or face-to-face dialogues that enable students to become aware of both their mistakes and what they are doing right in the way they express themselves.
- Use propositional calculus when presenting theoretical results in such a way as to ensure students become familiarised with their use.
- Undertake a periodic review together with students of the products involved in their work, in which the meanings of mathematical syntax, terms and symbols used in them are made explicit, in so doing checking the correct integration of natural language.
- Create accessible texts in the form of blogs that enable there to be interaction with the lecturer during the course.
- Provide students with back-up guides in order to help them prepare reports about the projects in which they are involved.
- Encourage students to submit their work for external review via competitions and/or publication.
- Encourage student participation in seminars, talks and events in order for them to give lectures or design posters about their work.
- Create workshops about common subjects.
- Encourage reading of texts about mathematics that enable there to be greater interaction with mathematical language.

5.2.4. *Strategies for assessment of results*

In this section we would do well to recall what was pointed out in the learning assessment strategies in the part referring to the general capacity being proposed, in which emphasis was placed on the importance of considering assessment as a formative process.

- Encouraging forms of communication among lecturers to provide them with support in fostering the development of competences.

- Jointly establishing assessment in levels of development for the learning results obtained by students such as: tasks, research reports, course notes and blogs. The language, content, logic in terms of formulation, ordering, structure and organisation, arguments, drawing of conclusions and their presentation should be taken into account.

- Encouraging the application of joint agreements in assessing results of written work and oral presentations.

5.2.5. *Example of course planning*

It is important to point out that the planning presented as an example in a given subject needs to be brought in line with the rest of subjects covered during the period subject to scrutiny (in this case, a semester) in each programme, and a student is deemed to have devoted themselves to their studies full time in order to ensure that this be feasible.

By way of an example of how teaching is organised for an undergraduate subject based on professional competences, Ordinary Differential Equations was chosen. This subject is taught during the first semester of the third year of the BA honours degree in Mathematics at the University of Havana. The total time set aside for the subject, according to the curriculum for this degree programme, is 96 contact teaching hours. This time is usually broken down into 6 hours a week over 16 weeks. However, in practice there may be semesters of 15 and even 14 weeks, which is why in this example 84 contact teaching hours are taken into consideration, of which 48 are set aside for lectures, 24 for joint practical work and 12 for seminars. Additionally, 12 consultations or tutorials were also included for dealing with students' doubts at times outside that scheduled for classes.

The assessment system for the subject consisted of the following activities:

1. Attendance by students in classes was recognized, along with their participation in the latter, mainly in theoretical-practical classes given jointly.

2. Surprise questions were asked in class of short duration (frequent control of student learning; three of these questions were prepared).

3. Guidance was provided in preparing a portfolio containing student answers to the new practical activities established throughout the semester (partial control of learning by chapter).

4. Exposés given in seminars over the last two weeks of the semester were assessed (partial control).

5. Students who did not respond to surprise questions or those who, having done so, provided unsatisfactory results, were able to attend a confirmatory meeting on completion of the fourteen weeks of class.

6. Final oral and written examination (final control of the subject). (The oral question is validated by participation in seminars).

7. Final assessment of the subject: consists of comprehensive assessment of all activities carried out by the student (frequent and partial control activities, in addition to the results of the final examination).

The total work time set aside by the student may be estimated as follows:

1. Individual study in the subject (text books and «Notes about Ordinary Differential Equations, prepared by the subject lecturer, Dr. Baldomero Valiño Alonso): 6 hours a week (1 hour per each hour of class). Total number of hours: 84.

2. Independent work involving dealing with problems contained in the practical activities attached to the subject: 5 hours per each

practical activity on average, which makes a total of 45 hours, since 9 practical activities were proposed.

3. Preparation of resolved problems in order to put together the subject portfolio (the «subject portfolio» must be submitted correctly without spelling or any other mistakes, preferably in latex, although it was accepted in manuscript form): 4 hours per each practical activity on average, 36 hours in total.

4. Preparation of exposés in seminars: 5 hours per each seminar, 10 hours in total (8 additional hours need to be added to include notes that students need to prepare for their presentations): 18 hours.

5. Preparation for the final examination: 8 hours a day over three or four days between an examination, 32 hours in total.

In short, distribution of total time set aside by students was estimated at 215 hours. Taking into account the credits system proposed by the Project, i.e. the Latin American Reference Credit (CLAR 2013) this subject would at the time have corresponded to between 7 and 8 CLAR credits; of course, student workload in this case was not assessed as a whole alongside the other subjects taught during the semester. As stated at the beginning, this is merely an example of an experience that might help to illustrate how much can be achieved in terms of developing competences – simply by carrying out typical activities that are normally undertaken at Latin American universities.

Below is shown how the control activities carried out in the subject enabled the development of generic and specific competences on the degree programme to be assessed:

1. Assessment of attendance in classes was based on a scale put together for the purpose of quantifying compliance with this essential requirement. This aspect enabled assessment of the generic competences of social responsibility and ethical commitment (both attached to the attitudinal dimension).

2. There were three partial surprise tests: one that lasted an entire period of class and a further two of shorter duration (around

15 minutes each). These activities enabled the assimilation of the generic competences to be verified such as «ability to learn and keep constantly up-to-date» (attitudinal dimension), «knowledge about the area of study and profession», «ability to apply knowledge in practice» and «ability to identify, consider and deal with problems» (cognitive and procedural dimension) and «capacity for oral and written communication» (relational dimension). Furthermore, these activities help to assess, among others, the following specific competences: «command of the basic concepts of higher mathematics» and «skill in quantitative reasoning» (cognitive and procedural dimension).

3. Assessment of how students deal with the exercises proposed in the series of nine «Practical Activities» aimed at individual practice and resolving problems encountered in the subject was carried out via a review of the portfolio submitted by each student. This aspect of assessment enabled assimilation of generic competences to be ascertained such as: «ethical commitment» and «ability to learn and keep constantly up-to-date » (attitudinal dimension); «capacity for abstraction, analysis and synthesis», «ability to apply knowledge in practice», «ability to identify, consider and deal with problems» (disciplinary dimension) and, among specific competences, the «willingness to deal with new problems in different areas» (attitudinal dimension), «ability to construct and develop logical arguments by clearly identifying hypotheses and conclusions» (disciplinary dimension); «ability to express oneself correctly using the language of mathematics» and «ability to use mathematical reasoning and draw relevant conclusions with clarity, accuracy and in a suitable manner for the target audience, both orally and in writing» (communicational dimension).

4. Assessment of individual participation in seminars on the subject (each student had to take part in two opportunities, as each team was assigned a couple of topics to explain). This aspect of assessment ascertained training in generic competences such as: «ability to learn and keep constantly up-to-date » (attitudinal dimension); «ability to apply knowledge in practice» and «ability to identify, consider and deal with problems» (disciplinary dimension); «capacity for oral and written communication» (communicational dimension) and «capacity for teamwork» (relational dimension). As for specific competences, basically the following were assessed: «ability to develop logical arguments

by clearly identifying hypotheses and conclusions» and «ability to help construct mathematical models from real phenomena» (disciplinary dimension); «capacity to express oneself correctly using the language of mathematics» and « ability to use mathematical reasoning and draw relevant conclusions with clarity, accuracy and in a suitable manner for the target audience, both orally and in writing» (communicational dimension) and «ability to work in interdisciplinary teams» (relational dimension).

5. The weighting of all these activities enabled the activity developed during the course by each student to be assessed. This assessment, together with the mark attained in the final examination (whether ordinary examination or resit) was taken into consideration in order to issue the final assessment for each student, based at all times on passing the final examination.

6. The results proved to be satisfactory, given that only one student failed the subject. That student is the only who from the group who has had to repeat the subject and did not take either the ordinary examination or the resit (they are an overseas scholarship holder).

7. The lecturer in charge of the subject drafted some «Notes about Ordinary Differential Equations», guides to carrying out the nine «Practical Activities», the «Guides to preparing seminars» and the «Guide to preparing the final examination» for the subject. These documents were sent by email to all students and which many (or even most) of them managed to print themselves and use extensively for independent study and for the development of all activities scheduled during the academic year. The publication of all notes prepared by way of a complement to the books provided to students can be proposed, as these were much more recently-updated. On different occasions when students were asked about this, they expressed satisfaction with these «Notes» and used them a great deal in order to orientate themselves in studying the subject as a complement to the teaching literature with which they were provided to help guide them.

8. The lecturer did lead on any of the topics during the seminars. Rather, these were prepared for and led by students via a study of the bibliography on the subject (including the «Notes» drafted

by the lecturer) and other literature recommended in the guide. For instance, the theory of equations and linear equation systems with variable coefficients (lecture No. 12 in «Notes about Ordinary Differential Equations») was set aside entirely for development in seminars; only the theory of equations and linear systems with constant coefficients was explained in class.

9. It should be stated that questions were included in the final examination about all topics covered by the analytical programme attached to the subject (irrespective of whether they had been covered or not by the lecturer in classes on the subject or had been topics developed by students in seminars).

Below are described the topics attached to the different activities that were carried out:

a) Lectures on the subject

1. Introduction to the theory of differential equations.
2. Elementary methods for integrating first-order ordinary differential equations: equations with separable variables.
3. Elementary methods for integrating first-order ordinary differential equations; equations that are reduced to equations with separable variables via transformations of coordinates in homogenous equations.
4. Elementary methods for integrating first-order ordinary differential equations: linear differential equations.
5. Elementary methods for integrating differential equations (final); exact differential equations; integrating factors. Do differential equations exist which cannot be integrated in quadratures?
6. Basic theorems about the theory of ordinary differential equations.
7. Approximate methods for resolving first-order ordinary differential equations: Cauchy's and Picard's theorems.

8. Basic theorems about existence and uniqueness for equations and normal systems of arbitrary-order ordinary differential equations.

9. Some examples of differential models.

10. Autonomous linear systems.

b) Topics covered by seminars over the last two weeks of the subject

Seminar 1: demonstration about basic theorems (existence and uniqueness, continuous dependence and differentiability of solutions with regard to initial conditions and parameters, theorems about theory of equations and linear equation systems with variable coefficients).

Seminar 2: demonstration about some theorems regarding the qualitative theory of ordinary differential equations and the stability theory; examples of applications of these theories.

c) Topics covered by «Practical Activities» in the subject

1. Basic concepts of first-order differential equations.

2. Equations with separable variables.

3. Equations that are reduced to equations with separable variables via transformations of coordinates; homogenous differential equations.

4. First-order linear equations; Bernouilli and Ricatti equations.

5. Exact differential equations; integrating factors; fields of directions defined by grade 1 differential forms in the plane.

6. Some applications of first-order ordinary differential equations.

7. Equations and linear differential equation systems with constant coefficients.

8. Equations and linear ordinary differential equation systems with variable coefficients.

9. Stability of solutions to ordinary differential equation systems.

d) Assessment activities carried out in the subject (frequent, partial and final)

Assessment activities:

1. Surprise question 1: elementary properties of first-order ordinary differential equations.

2. Surprise question 2: Cauchy-Picard's theorem; successive approximations of Picard's theorem.

3. Surprise question 3: theorem about the solution to Cauchy's problem for a linear equation system with constant coefficients.

4. Confirmatory meeting during the last week of classes; miscellaneous topics pursued in the subject throughout the semester.

5. Assessment of presentations by teams in seminars (all students from each team intervened in exposés).

6. Periodic assessment of portfolios submitted by students.

7. Ordinary final examinations and resits.

The most outstanding students from the year (i.e. those that obtained the highest assessment in the subject) were able to sit what is referred to as an *exámen de prueba* or *honours examination* in the first semester of the following year. This examination was based on preparation of an essay about any of the three topics about which students had been previously oriented, and which they chose independently and submitted in pdf format for the lecturer to assess.

The topics proposed to students for essays were as follows:

1. Differential equations over differentiable varieties: phase flow defined by a vector field.

2. Main results of the qualitative theory of ordinary differential equations.

3. Applying the theory of ordinary differential equations in different problems: theory of prey-predator coexistence, mathematical theories of war and theory of epidemics, etc.

As can be noted, the topics proposed did not form part of the syllabus, although any of them could be prepared based on the basic results obtained during the year. Students were recommended basic literature, although they were completely free to use those works that best suited their needs and, in fact, this is what they did.

The following aspects were taken into consideration when assessing the essays submitted by each student:

1. Spelling.
2. Quality of writing and adherence to the maximum number of pages required (10 pages).
3. Clarity in explaining the topic chosen.
4. Reasoning and demonstrations.
5. The calculations made.
6. Adherence of the essay to the topics proposed.
7. Importance of the topic in applying the theory of ordinary differential equations.

All students submitted interesting essays that clearly showed their ability to learn about new topics independently that they had not previously studied while learning the subject, and were able to refer or even explicitly demonstrate major applications for their use.

This enabled the generic and specific competences attained by the most outstanding students from the school year to be assessed at the highest level, these competences including – among others – the following: «capacity for abstraction, including the logical development of mathematical theories and relationships between them», «ability to embark on mathematical research under expert guidance», «ability to help construct mathematical models from real situations», «ability to express oneself correctly using the language of mathematics», «ability to use mathematical reasoning and draw relevant conclusions with clarity, accuracy and in a suitable manner for the target audience» and «knowledge of the English language», specifically for the purpose of consulting the bibliography existing in that language.

5.2.6. *Example of a learning environment*

A learning environment is a meeting point for lecturers and students that requires their active participation, this being understood as referring to the development of activities that involve reflection, interaction and cooperation. It should nurture situations in which students may find the meanings of mathematical concepts and skills that include the reasons for courses of action within their context. The inclusion of technological tools does not in itself guarantee significant learning, but can be used to reorganise and extend meanings about what is being learned both synchronously and asynchronously and to foster collaborative work.

At present, it is noted that the lecturer's role is changing – from providing information, controlling and ensuring uniformity to creating learning situations, challenging, lending support and diversifying. Virtual environments lend support to and enrich interaction on the part of each student with their classmates and lecturers. In this area, everyone integrates theories and technologies with texts and hypertexts, which gives rise to a permanent construction process. The pedagogical role of the lecturer intervening in this process is important in addition to synchronous and asynchronous action in which use is made of texts, audio and video.

The activities proposed in the virtual environment include reading about topics of study in class in order to establish moments of construction or reconstruction and systematisation of concepts via discussions about key ideas. In these discussions are assessed the pooling together of ideas, interaction and collaboration, as well as the possibility of transforming mistakes into opportunities that can be developed via skills such as analysis and clear reasoning based on the theories that have been studied, so as to defend their points of view, explain them and develop their own ideas. Some of these skills will be used by graduates in the responsibilities they will assume in their future careers.

The example that has been summarised was put into practice at Universidade de Caxias do Sul and referred to the subject *Differential and Integral Calculus*. There was a section devoted to the presentation of the platform, while other sections were devoted to providing information regarding syllabuses and to schedules of activities and initial guidelines for use of the platform, suggestions about bibliography, links to sites related to the topics being studied and back-up material such as classroom notes. Attention should be drawn to the following contexts in the virtual environment:

Discussions

There is an area in which constriction and interaction take place in a forum, which enables attached files to be used that are essential for using the language of mathematics that otherwise might not be found in editors of common texts. A topic for discussion is registered in this area, in which the name of the person who proposes it, the responses and the date are included. Identifying difficulties, improving comprehension, establishing doubts and pooling them together and encouraging benefits for the whole group involved is made possible in these discussions, all the time there is interest in doing so. A discussion may deal with the analysis of a rational function whose numerator is $x - 3$ and with a denominator $x + 2$, and may start from wording as follows: is it correct to state that the only point of intersection on the graph for the function with coordinate axes is $x = 3$?

Mathematical software

One option is to include libraries devoted to algebraic, geometric and numeric processing that offer the chance to use mathematical syntax as it is usually expressed which might or might not include knowledge of programming. In it appear instructions for installing the software and examples of what this software is able to help develop that is related to the course.

Activities

This is a section devoted to the registration of all organised activities that are encouraged in the form of hypertexts that contain everything produced, for instance:

- T1 Collaborative work: functions topic.
- T2 Collaborative work: functions topic; limits and continuity.
- T3 Work regarding the first partial assessment; work guidelines for the first assessment.
- T4 Collaborative work about polynomial functions: activity 1.
- T5 Collaborative work about rational functions: activity 2.
- T6 Collaborative work about trigonometric functions: activity 3.

A portfolio of each student is included in the activities section, the aim of which is to act as a register for what they produce and any other materials resulting from their study.

Individual participation

This enables the lecturer to monitor and provide feedback to each student. This shows whether they carried out the activity or not, whether they did so partially or whether they did so entirely. Contact on the part the lecturer is always informative in nature in the form of an invitation to reflect on questioning or not.

Group production

Each group production of work takes place during discussions related to each activity in which all interventions are concentrated. Discussions about a specific activity are transferred to this section once the activity being encouraged has been completed.

6

Conclusions

The education and training of new professionals demands the confluence of experiences and their analysis in order to encourage individuals to develop appropriate knowledge and be able to develop their potential. For its part, mathematics requires qualification in the subject in order for it to be learned, although relating mathematical concepts to real situations is also necessary.

It is the duty of universities to educate creative citizens to help them embark on new projects and acquire skills, so as to enable them in turn to deduce, organise, compare, interpret and establish relations. How students express themselves enables one to observe that the purpose of learning and conduct apparent in the student-lecturer relationship needs to be reviewed and transformed in accordance with the standpoint of what is required in order to develop society.

The Tuning Project in Latin America has enabled reflection on the part of its participants to be extended to teachers' associations at the different universities.

One major aspect was that, to obtain the results established herein, teaching staff were involved in the response to different questionnaires by means of which the following was made possible: the establishment of the meta-profile, its contrasting with current profiles, the degree of appropriation of the different competences, the collective exercise that involved analysing and explaining teaching, learning and assessment strategies for competences, and reflection with a view to ensuring that students appreciate what their daily work implies. Participation on the part of students in responding to questionnaires about an estimate of the time set aside for pursuing their studies proved also to be of importance.

In all cases, a positive and purposeful reflection was inferred in order to envisage the responsibilities attached to lecturers in developing the curriculum - for students, this meant being aware of aspects that demand their attention.

Furthermore, the meta-profile and its minimum content in several of the participant universities have already proved useful in reaffirming their curricular plans and in warning of the need for changes.

Without doubt, communication and collaborative networks have been strengthened and extended as a result of disseminating the project to other parts of the world. The possibilities deriving from meetings and the media enabled the different regions that took part in the project to learn about and understand the various work and approaches.

Another aspect that should be pointed out is the possibility of sharing and learning throughout the course of the project about the reports on each of the topics covered in the different disciplinary areas, which evidence a range of approaches, features, methodologies and analyses.

This represented a great opportunity for collaborative work on the part of participants in the area and also the chance to strengthen friendly relations.

7

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8

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