Tuning

Tuning Educational Structures in Europe

Reference Points for the Design and Delivery of Degree Programmes in **Mathematics**

Education and Culture DG
Life Long Learning

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The name *Tuning* was chosen for the project to reflect the idea that universities do not look for uniformity in their degree programmes or any sort of unified, prescriptive or definitive European curricula but simply for points of reference, convergence and common understanding. The protection of the rich diversity of European education has been paramount in the Tuning Project from the very start and the project in no way seeks to restrict the independence of academic and subject specialists, or undermine local and national academic authority.

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Content

Introduction to the Tuning project

Tuning Educational Structures in Europe is a university driven project which aims to offer a universal approach to implement the **Bologna Process** at the level of higher education institutions and subject areas. The Tuning approach consists of a methodology to (re-) design, develop, implement and evaluate study programmes for each of the Bologna cycles.

Furthermore, Tuning serves as a platform for developing reference points at subject area level. These are relevant for making programmes of studies comparable, compatible and transparent. Reference points are expressed in terms of learning outcomes and competences. Learning outcomes are statements of what a learner is expected to know, understand and be able to demonstrate after completion of a learning experience According to Tuning, learning outcomes are expressed in terms of the *level of competence* to be obtained by the learner. Competences represent a dynamic combination of cognitive and meta-cognitive skills, knowledge and understanding, interpersonal, intellectual and practical skills, and ethical values. Fostering these competences is the object of all educational programmes. Competences are developed in all course units and assessed at different stages of a programme. Some competences are subject-area related (specific to a field of study), others are generic (common to any degree course). It is normally the case that competence development proceeds in an integrated and cyclical manner throughout a programme. To make levels of learning comparable the subject area groups/Thematic Networks have developed cycle (level) descriptors which are also expressed in terms of competences.

According to Tuning, the introduction of a three cycle system implies a change from a staff centred approach to a student oriented approach. It is the student that has to be prepared as well as possible for his or her future role in society. Therefore, Tuning has organized a Europe-wide consultation process including employers, graduates and academic staff / faculty to identify the most important competences that should be formed or developed in a degree programme. The outcome of this consultation process is reflected in the set of reference points – generic and subject specific competences – identified by each subject area.

Besides addressing the implementation of a three cycle system, Tuning has given attention to the Europe-wide use of the student workload based European Credit Transfer and Accumulation System (ECTS). According to Tuning ECTS is not only a system for facilitating the mobility of students across Europe through credit accumulation and transfer; ECTS can also facilitate programme design and development, particularly with respect to coordinating and rationalising the demands made on students by concurrent course units. In other words, ECTS permits us to plan how best to use students' time to achieve the aims of the educational process, rather than considering teachers' time as a constraint and students' time as basically limitless. According to the Tuning approach credits can only be awarded when the learning outcomes have been met.

The use of the learning outcomes and competences approach might also imply changes regarding the teaching, learning and assessment methods which are used in a programme. Tuning has identified approaches and best practices to form specific generic and subject specific competences.

Finally, Tuning has drawn attention to the role of quality in the process of (re-)designing, developing and implementing study programmes. It has developed an approach for quality enhancement which involves all elements of the learning chain. It has also developed a number of tools and has identified examples of good practice which can help institutions to boost the quality of their study programmes.

Launched in 2000 and strongly supported, financially and morally, by the European Commission, the Tuning Project now includes the vast majority of the Bologna signatory countries.

The work of Tuning is fully recognized by all the countries and major players involved in the Bologna Process. At the Berlin Bologna follow-up conference which took place in September 2003, degree programmes were identified as having a central role in the process. The conceptual framework on which the Berlin Communiqué is based is completely coherent with the Tuning approach. This is made evident by the language used, where the Ministers indicate that degrees should be described in terms of workload, level, learning outcomes, competences and profile.

As a sequel to the Berlin conference, the Bologna follow-up group has taken the initiative of developing an overarching *Framework for Qualifications of the European Higher Education Area* (EQF for HE) which, in concept and language, is in full agreement with the Tuning approach.

This framework has been adopted at the Bergen Bologna follow-up conference of May 2005. The EQF for Higher Education has made use of the outcomes both of the Joint Quality Initiative (JQI) and of Tuning. The JQI, an informal group of higher education experts, produced a set of criteria to distinguish between the different cycles in a broad and general manner. These criteria are commonly known as the "*Dublin descriptors*". From the beginning, the JQI and the Tuning Project have been considered complementary. The JQI focuses on the comparability of cycles in general terms, whereas Tuning seeks to describe cycle degree programmes at the level of subject areas. An important aim of all three initiatives (EQF, JQI and Tuning) is to make European higher education more transparent. In this respect, the EQF is a major step forward because it gives guidance for the construction of national qualification frameworks based on learning outcomes and competences as well as on credits. We may also observe that there is a parallel between the EQF and Tuning with regard to the importance of initiating and maintaining a dialogue between higher education and society and the value of consultation -- in the case of the EQF with respect to higher education in general; in that of Tuning with respect to degree profiles.

In the summer of 2006 the European Commission launched a European Qualification Framework for Life Long Learning. Its objective is to encompass all types of learning in one overall framework. Although the concepts on which the EQF for Higher Education and the EQF for LLL are based differ, both are fully coherent with the Tuning approach. Like the other two, the LLL variant is based on the development of level of competences. From the Tuning perspective both initiatives have their value and their roles to play in the further development of a consistent European Education Area.

This brochure reflects the outcomes of the work done by the Subject Area Group (SAG) Mathematics so far. The outcomes are presented in a template that was developed to facilitate readability and rapid comparison across the subject areas. The summary aims to provide, in a very succinct manner, the basic elements for a quick introduction into the subject area. It shows in synthesis the consensus reached by a subject area group after intense and lively discussions in the group. The more ample documents on which the template is based are also included in the brochure. They give a more detailed overview of the elaborations of the subject area group.

The Tuning Management Committee

Mathematics

This brochure summarises the work of the Tuning Mathematics group in the projects Tuning I to IV revised after the validation conference of 2007. Much of the material has appeared previously in the reports of Tuning I and II[1] and is also available on the Tuning project website.[2] A document which proved extremely useful in this process and which met with unanimous agreement from the group was the UK Quality Assurance Agency Benchmark document on Mathematics, Statistics and Operational Research[3]. It is quoted from almost verbatim at some points in this document.

1 Final reports Tuning I and Tuning II, Julia Gonzalez and Robert Wagenaar (eds), 2002 and 2005 (resp).

2 http://tuning.unideusto.org/tuningeu/

3 http://www.qaa.ac.uk/crntwork/benchmark/phase2/mathematics.pdf

1. Mathematics as a discipline

Mathematics as an intellectual discipline traces its roots back through all the major civilisations to the earliest recorded human works. While it originated as a systematisation of the solutions of practical problems in areas such as land surveying (hence geometry), construction, war and commerce, it evolved with the realisation that abstraction of the essentials led to generalisation of the applications and hence became a science which uses rigorous deduction to arrive at solid conclusions from clearly stated assumptions. It is a discipline that requires people to think with logic, criticism, rigour and depth.

Its abstraction makes it applicable to almost any discipline, since it identifies patterns that are common to many areas, such as health sciences, earth sciences, astronomy, computer sciences, etc.

Mathematics is fundamental not only to much of science and technology but also to almost all situations that require an analytical model-building approach, whatever the discipline. In recent decades there has been an explosive growth of the use of mathematics in areas outside the traditional base of science, technology and engineering.

Statistics as a discipline within mathematics arose from probability, which seemingly originated from considerations of gambling and developed further in the nineteenth century with the growth of "official statistics". It is the science of modelling random phenomena and making inferences. It uses mathematical techniques and ideas to solve problems involving randomness, chance, variability, risk and so on. Statistics plays a major and increasing role in personal and public life, particularly in medicine, quality control and management, all areas of physical and social sciences, business and economics.

Programmes in mathematics can and do vary from the very pure or theory-based to the very applied or practice based. Some are broad, while others allow specialisation in particular areas, such as statistics or financial mathematics. They all share the key learning outcomes detailed in what follows.

2. Degree profiles

Typical degrees offered

— First cycle in

- Mathematics
- Applied Mathematics
- Mathematical Sciences
- Mathematical Physics
- Mathematics and Statistics
- Financial Mathematics

— Second cycle in

- Mathematics
- Statistics
- Financial Mathematics
- Biomathematics

— Third cycle in any specialised area of Mathematics

3. Typical occupations of graduates

First cycle

Programme Profile	Category / Group of professions	Example professions
Mathematics with Education	Education	Secondary school teacher of Mathematics
Mathematics specialising in statistics	Industry/Government/ Health sector	Statistician
Mathematics possibly specialising in finance, statistics or economics	Banking/Insurance/ Business	Actuary, Banker, Risk Manager, Accountant
Mathematics with significant computer science	Industry/Banking	Software analyst

Second cycle

Programme profile	Category / Group of professions	Example professions
Mathematics specialising in statistics at second cycle	Industry/Government/ Health sector	Statistician
Mathematics specialising in finance, statistics or econometrics at second cycle	Banking/Insurance/ Business	Actuary, Banker, Risk Manager, Accountant
Mathematics with specialisation at second cycle	Industry/Defence Industry	Researcher

Third cycle

Programme profile	Category / Group of professions	Example professions
Any specialised field of Mathematics	University	Researcher/Teacher
Statistics	Industry, in particular biotechnology and medicine	Researcher
Financial or actuarial mathematics	Banking/Insurance/ Business	Actuary, Banker, Risk Manager
Algebra	Government	Researcher, Cryptologist

4. Role of mathematics in other degree programmes

Mathematics is an essential component of all engineering and most science courses, in particular physics, but also chemistry and, increasingly, biology. Some mathematics units are included in most courses in business studies and economics; statistics has particular importance in these areas and also in programmes in the humanities where no other mathematics courses may form part of the programme.

It also commonly occurs as one subject in a two subject degree, such as Mathematics and Economics, Mathematics and Computer Science, Mathematics and Biology, Mathematics and Physics, Mathematics and Economics, Mathematics and Education. This list is not exhaustive, and many variations of the listed titles exist, while more will probably emerge, reflecting Mathematics universal relevance. While no generalisations can be made, the ordering of a subject area in the title may reflect its relative weight. This document focuses mainly on degrees in Mathematics, so we merely note here that a first cycle interdisciplinary degree would need to have a significant proportion of mathematical content to enable progression to a second cycle programme in mathematics.

5. Consultation process with stakeholders

Graduates and employers were consulted about generic and some subject specific competences in a questionnaire as part of Tuning I and again as part of Tuning IV; academics were consulted also about more detailed subject specific competences. The results informed the subsequent paper "Towards a common framework for Mathematics degrees in Europe", which was also published in the European Mathematical Society Newsletter in September 2002 (pp. 26-28)[4]. This paper has also been widely disseminated at national levels and is substantially reproduced in section 6.

4 http://www.ems-ph.org/journals/newsletter/pdf/2002-09-45.pdf

6. Towards a common framework for Mathematics degrees in Europe

6.1. A common framework: what it should and should not be or do

6.1.1. The only possible aim in agreeing a "common European framework" should be to facilitate the automatic recognition of mathematics degrees in Europe in order to help mobility. By this we mean that when somebody with a degree in mathematics from country A goes to country B:

a) He/she will be legally recognised as holding such a degree, and the Government of country B will not require further proof of competence.

b) A potential employer in country B will be able to assume that he/she has the general knowledge expected from somebody with a mathematics degree.

Of course, neither of these guarantees employment: the mathematics graduate will still have to go through whatever procedures (competitive exams, interviews, analysis of his/her curriculum, value of the degree awarding institution in the eyes of the employer...) are used in country B to get either private or public employment.

6.1.2. One important component of a common framework for mathematics degrees in Europe is that all programmes have similar, although not necessarily identical, structures. Another component is agreeing on a basic common core curriculum while allowing for some degree of local flexibility.

6.1.3. We should emphasise that by no means do we think that agreeing on any kind of common framework can be used as a tool for automatic transfer between Universities. These will always require consideration by case, since different programmes can get students to adequate levels in different but coherent ways, but an inappropriate mixing of programmes may not.

6.1.4. In many European countries there exist higher education institutions that differ from universities both in the level they demand from students and in their general approach to teaching and learning. In fact, in order not to exclude a substantial number of students from higher education, it is essential that these differences be maintained. We want to make explicit that this paper refers only to universities (including technical universities), and that any proposal of a common framework designed for universities would not necessarily apply to other types of institutions.

6.2. Towards a common core mathematics curriculum

6.2.1. General remarks

At first sight, mathematics seems to be well suited for the definition of a core curriculum, e.g. for the first two or three years. Because of the very nature of mathematics, and its logical structure, there will be a common part in all mathematics programmes, consisting of the fundamental notions. On the other hand, there are many areas in mathematics, and many of them are linked to other fields of knowledge (computer science, physics, engineering, economics, etc.). Flexibility is of the utmost importance to keep this variety and the interrelations that enrich our science.

There could possibly be an agreement on a list of subjects that must absolutely be included (linear algebra, calculus/analysis) or that should be included (probability/statistics, some familiarity with the mathematical use of a computer) in any mathematics degree. In the case of some specialised courses, such as mathematical physics, there will certainly be variations between countries and even between universities within one country, without implying any difference of quality of the programmes.

Moreover, a large variety of mathematics programmes exist currently in Europe. Their entry requirements vary, as do their length and the demands on the student. It is extremely important that this variety be maintained, both for the efficiency of the education system and socially, to accommodate the possibilities of more potential students. To fix a single definition of contents, skills and level for the whole of European higher education would exclude many students from the system, and would, in general, be counterproductive.

In fact, the group is in complete agreement that programmes could diverge significantly beyond the basic common core curriculum (e.g. in the direction of "pure" mathematics, or probability - statistics applied to economy or finance, or mathematical physics, or the teaching of mathematics in secondary schools). The presentation and level of rigour, as well as accepting there is and must continue to be variation in emphasis and, to some extent, content, even within the first two or three years, will make all those programmes recognisable as valid mathematics programmes.

As for the second cycle, not only do we think that programmes could differ, but we are convinced that, to reflect the diversity of mathematics and its relations with other fields, all kinds of different second cycles in mathematics should be developed, using in particular the specific strengths of each institution.

6.2.2. The need for accreditation

The idea of a basic core curriculum must be combined with an accreditation system. If the aim is to recognise that various universities fulfil the requirement of the core curriculum, then one has to check on three aspects:

— a list of contents

— a list of skills

— the level of mastery of concepts

These cannot be reduced to a simple scale. To give accreditation to a mathematics programme, an examination by a group of peer reviewers, mostly mathematicians, is necessary. The key aspects to be evaluated should be:

a) the programme as a whole

b) the units in the programme (both the contents and the level)

c) the entry requirements

d) the learning outcomes (skills and level attained)

e) a qualitative assessment by both graduates and employers

The group does not believe that a (heavy) system of European accreditation is needed, but that universities in their quest for recognition will act at the national level. For this recognition to acquire international standing, the presence on the review panel of mathematicians from other countries seems essential.

6.3. Some principles for a common core curriculum for the first degree (Bachelor) in mathematics

We do not feel that fixing a detailed list of topics to be covered is necessary, or even convenient. But we think that it is possible to give some guidelines as to the common contents of a "European first degree in mathematics", and more important, as to the skills that all graduates should develop.

6.3.1. Contents

6.3.1.1. All mathematics graduates will have knowledge and understanding of, and the ability to use, mathematical methods and techniques appropriate to their programme. Common ground for all programmes will include calculus in one and several real variables, and linear algebra.

6.3.1.2. Mathematics graduates must have knowledge of the basic areas of mathematics, not only those that have historically driven mathematical activity, but also others of more modern origin. Therefore graduates should normally be acquainted with most, and preferably all, of the following: language of elementary set theory, basic differential equations; basic complex functions; some probability; some statistics; some numerical methods and computer simulation; basic geometry of curves and surfaces; some algebraic structures; some discrete mathematics; and some modelling from a related discipline.

6.3.1.3. These need not be learned in individual modules covering each subject in depth from an abstract point of view. For example, one could learn about groups in a course on (abstract) group theory or in the framework of a course on cryptography. Geometric ideas, given their central role, could appear in a variety of courses.

6.3.1.4. Other methods and techniques will be developed according to the requirements and character of the programme, which will also largely determine the levels to which the developments are taken. In any case, all programmes should include a substantial number of courses with mathematical content.

6.3.1.5. In fact, broadly two kinds of mathematics curricula currently coexist in Europe, and both are useful. Let us call them, following the QAA Benchmark document, "theory based" and "practice based" programmes. The weight of each of the two kinds of programmes varies widely depending on the country, and it might be interesting to find out whether most European university programmes of mathematics are "theory based" or not.

Graduates from theory-based programmes will have knowledge and understanding of results from a range of major areas of mathematics. Examples of possible areas are algebra, analysis, geometry, number theory, differential equations, mechanics, probability theory and statistics, but there are many others. This knowledge and understanding will support the knowledge and understanding of mathematical methods and techniques, by providing a firmly developed mathematical context.

Graduates from practice-based programmes will also have knowledge of results from a range of areas of mathematics, but the knowledge will commonly be designed to support the understanding of models and how and when they can be applied. Besides those mentioned above, these areas include numerical analysis, control theory, operations research, discrete mathematics, game theory and many more. (These areas may of course also be studied in theory-based programmes.)

6.3.1.6. It is necessary that all graduates will have met at least one major area of application of their subject in which it is used in a serious manner and this is considered essential for a proper appreciation of the subject. The nature of the application area and the manner in which it is studied might vary depending on whether the programme is theory-based or practice based. Possible areas of application include physics, astronomy, chemistry, biology, engineering, computer science, information and communication technology, economics, accountancy, actuarial science, finance and many others.

6.3.2. Skills

6.3.2.1. For a standard notion like integration in one variable, the same "content" could imply:

— computing simple integrals

— understanding the definition of the Riemann integral

— proving the existence and properties of the Riemann integral for classes of functions

— using integrals to model and solve problems of various sciences.

So, on one hand the contents must be clearly spelled out, and on the other various skills are developed by the study of the subject.

6.3.2.2. Students who graduate from programmes in mathematics have an extremely wide choice of career available to them. Employers greatly value the intellectual ability and rigour and the skills in reasoning that these students will have acquired, their firmly established numeracy, and the analytic approach to problem-solving that is their hallmark.

Therefore, the three key skills that we consider may be expected of any mathematics graduate are:

a) the ability to conceive a proof,

b) the ability to model a situation mathematically,

c) the ability to solve problems using mathematical tools.

It is clear that, nowadays, solving problems should include their numerical and computational resolution. This requires a sound knowledge of algorithms and programming and the use of the existing software.

6.3.2.3. Note also that skills and level are developed progressively through the practice of many subjects. We do not start a mathematics programme with one course called "how to make a proof" and one called "how to model a situation", with the idea that those skills will be acquired immediately. Instead, it is through practice in all courses that these develop.

6.3.3. Level

All graduates will have knowledge and understanding developed to higher levels in particular areas. The higher-level content of programmes will reflect the title of the programme. For example, graduates from programmes with titles involving statistics will have substantial knowledge and understanding of the essential theory of statistical inference and of many applications of statistics. Programmes with titles such as mathematics might range quite widely over several branches of the subject, but nevertheless graduates from such programmes will have treated some topics in depth.

6.4. The second degree (Master) in mathematics

We have already made explicit our belief that establishing any kind of common curriculum for second cycle studies would be a mistake. Because of the diversity of mathematics, the different programmes should be directed to a broad range of students, including in many cases those whose first degree is not in mathematics, but in more or less related fields (computer science, physics, engineering, economics, etc.). We should therefore aim for a wide variety of flavours in second cycle programmes.

Rather than the contents, we think that the common denominator of all second cycles should be the level of achievement expected from students. A unifying characteristic feature could be the requirement that all second cycle students carry out a significant amount of individual work. This could be reflected in the presentation of a substantial individual project.

We believe that, to be able to do real individual work in mathematics, the time required to obtain a Master's qualification should be the equivalent of at least 90 ECTS credits. Therefore, depending on the national structure of first and second cycles, a Master would typically vary between 90 and 120 ECTS credits.

6.5. A common framework and the Bologna agreement

6.5.1. How various countries implement the Bologna agreement will make a difference on core curricula. In particular, 3+2 may not be equivalent to 5, because, in a 3+2 years structure, the 3 years could lead to a professional diploma, meaning that less time is spent on fundamental notions, or to a supplementary 2 years, and in that case the whole spirit of the 3 years programme should be different.

6.5.2. Whether it will be better for mathematics studies to consist of a 180 ECTS Bachelor, followed by a 120 ECTS Master (a 3+2 structure in terms of academic years), or whether a 240+90 (4+1+project) structure is preferable, may depend on a number of circumstances. For example, a 3+2 break up will surely facilitate crossing between fields, where students pursue Masters in areas different from those in which they obtained their Bachelor degree.

One aspect that cannot be ignored, at least in mathematics, is the training of secondary school teachers. If the pedagogical qualification must be obtained during the first cycle studies, these should probably last for 4 years. On the other hand, if secondary school teaching requires a Master (or some other kind of postgraduate qualification), a 3 years Bachelor may be adequate, with teacher training being one of the possible postgraduate options (at the Master's level or otherwise).

6.5.3. The group did not attempt to solve contradictions that could appear in the case of different implementations of the Bologna agreement (i.e. if three years and five years university programmes coexist; or different cycle structures are established: 3+1, 3+2, 4+1, 4+1+project, 4+2 have all been proposed). As we said before, it might be acceptable that various systems coexist, but we believe that large deviations from the standard (such as a 3+1 structure, or not following the principles stated in section 3) need to be grounded in appropriate entry level requirements, or other programme specific factors, which can be judged by external accreditation. Otherwise, such degrees risk not benefiting from the automatic European recognition provided by a common framework, even though they may constitute worthy higher education programmes.

6.6. Doctorates in Mathematics

Currently, throughout Europe, doctorates usually comprise research under the guidance of a thesis advisor or supervisor, sometimes also including some coursework. While credits are awarded for the coursework, it is unusual for credits to be awarded for the research. There are various funding mechanisms in place, most requiring students to act as teaching assistants. The duration of studies seems to be, on average, between 3 and 4 years, although in some countries it is typically longer; duration is affected by the nature of the funding and also the by the structure of the doctoral programme.

While coursework is in some countries taken into account, most commonly the award of a doctorate is based on a defence of a thesis. In some countries the award is graded. Theses are usually in the native language of the country concerned, but English is also commonly used, or a summary of the thesis may appear in English.

A practice-based doctorate will not necessarily require the proof of a theorem; nor will doctorates in, for example, Mathematical Education or the History of Mathematics. However the unifying characteristic feature should be the requirement that all doctoral candidates carry out a significant piece of original research. The group felt that, while ECTS credits are appropriate to the coursework elements of a doctoral programme, they serve no purpose with respect to the research work, other than in a purely formal sense.

7. Workload and ECTS

7.1. ECTS

While most of the European higher education area is tending towards 180 ECTS first cycle degrees, Spain, Portugal and Ireland, for various reasons, not least the age of entry, will most likely have a preponderance of 240 ECTS first cycle programmes. The Tuning group expressed the view that if a paedagogical qualification were to be obtained as part of the first cycle, then it should have 240 ECTS credits. The requirement that a thesis or dissertation form a significant part of the second cycle suggests a range of 90 to 120 credits at this cycle.

7.2. Planning form for a sample module

The following example of a first cycle course for first semester students in Mathematics and Physics is based on a semester with 14 weeks and a workload of 900 hours per semester so that 1 ECTS credit corresponds to a workload of 30 hours.

It is a typical core curriculum course unit in analysis where the last two weeks repeat and condense some results of the first 12 weeks on an advanced level and serve as foundation for analysis of functions of several variables.

There are weekly 2 lectures (2 times 1.5 hours) and 1 problems class (1 times 1.5 hours). Thus the presence time per week in the lecture room is

$$14 \text{ (weeks) } (4.5 \text{ hours}) = 63 \text{ hours.}$$

In the first week of the semester there is a lecture instead of the problems class. Moreover the last two problems classes (week 13 and 14) serve as preparation for the final test, which takes place within three weeks after the end of the semester. Weekly homework is mandatory, the homework will be corrected each week, and it is necessary to have at least half of correct solutions to be admitted to the final test which lasts three hours. If the students get at least half of the maximum of available points in the final test they get 10 ECTS credits.

The calculation for the workload is as follows:

Presence time in the lecture room	63 h
Presence time for the final test	3 h
Study of the weekly lecture = 8 h (thus 14 (8 = 112 h)	112 h
Weekly time for the solution of the 11 problems = 10 h (thus 11 (10 = 110 h)	110 h
Preparation for the final test	12 h
Sum	300 h

Programme of Studies:	Beginners Course
Name of module/course unit:	Differentiation and Integration
Target group:	First cycle students in Mathemat ics, Physics
Level of the module/course unit:	Bachelor level 1
Number of ECTS credits:	10 (workload is 300 hours; 1 credit = 30 hours)

Competences to be developed:

1. Profound knowledge of basic techniques in the theory of series, differentiation and integration
2. Understanding the principles that provide these basic techniques
3. Knowledge and understanding of logical and deductive arguments
4. Capacity for using formal arguments and notations in mathematical proofs
5. Capacity for localisation of assumptions within the proofs of Theorems
6. Capacity for finding rigorous proofs of small problems
7. Development of capacity using methods of analysis to problems in applications

Learning outcomes	Educational activities	Estimated student work time in hours
week 1		
To understand the basic concepts of logic that underlie all mathematical reasoning. Calculating truth tables. Translating verbal expressions into logical terms and conversely	Lecture 1 Logical operators, logical equivalence and logical consequence. Basic notations of set theory. Negation of statements with quantifiers	1.5
	Study of lecture 1	4
Familiarity with applying the principle of induction with binomial coefficients and with Pascal's triangle.	Lecture 2 Short axiomatic introduction of **N, Z, Q, R**. The principle of induction. Binomial Theorem. **R** as an ordered field.	1.5
Familiarity with manipulation of inequalities. Knowledge about the difference of **Q** and **R.**	Lecture 3 (=Problems class) Absolute value. Inequalities. Supremum and infimum of subsets of **R.** Axiom of Archimedes. Axiom of completeness. Density of **Q** in **R**.	1.5
	Study of Lectures 2 and 3	4
week 2		
To understand the concept of convergence in epsilon-N notation To learn the difference between limits and limit points. To be able to apply limit theorems	Lecture 4 Convergence of sequences. Limit points. Theorem of Bolzano-Weierstrass. Algebra of limit theorems. Limit superior and inferior.	1.5
	Study of lecture 4	4
Knowledge about connection between sequences and series. Calculation of elementary series.	Lecture 5 Cauchy sequences. Completeness of **R**. Definition of powers. Series and examples of series.	1.5
	Study of lecture 5	4
	Problems class 1	1.5
	Time for homework 1	10

Learning outcomes	Educational activities	Estimated student work time in hours
week 3		
To be able to apply the different tests for convergence (ratio, alternating, *n*th root test). Practical calculation of the Cauchy product.	*Lecture 6* Conditional and absolute convergence. Tests for convergence of series. Operations involving series. Cauchy product.	1.5
	Study of lecture 6	4
Familiarity with the e-d notation of limits of functions.	*Lecture 7* Functions and graphs. Operations of functions. Injective, surjective, inverse functions. Monotonicity, boundedness and limits of functions	1.5
	Study of lecture 7	4
	Problems class 2	
	Time for homework 2	
week 4		
Calculation of limits of functions. Decision (by arguments) whether a function is continuous or not.	*Lecture 8* Limit theorems for functions. Continuity. The intermediate value theorem. Uniform continuity.	1.5
	Study of lecture 8	4
To learn the importance of uniform convergence concerning interchanging of limit processes. To decide whether a sequence or series of functions is pointwise or uniformly convergent (or not).	*Lecture 9* Pointwise and uniform convergence of functions. Interchange of limit and continuity. Cauchy Criterion for sequences and series of functions. Weierstrass M-test.	1.5
	Study of Lecture 9	4
	Problems class 3	1.5
	Time for homework 3	10

Learning outcomes	Educational activities	Estimated student work time in hours
week 5		
Calculation of the interval of convergence. To learn the importance of limes superior.	*Lecture 10* Power series. Radius of convergence. Properties of power series. Series expansion for e^x	1.5
	Study of Lecture 10	4
Deduction of trigonometrical identities from the addition theorems for sine and cosine.	*Lecture 11* Cauchy functional equations. Introduction of $x \to cx$, c^x, $c.\ln x$, x^c Introduction of sine and cosine by power series. Addition theorems.	1.5
	Study of Lecture 11	4
	Problems class 4	1.5
	Time for homework 4	10
week 6		
Knowing the graphs and properties of all elementary circular-, area-functions and their inverses	*Lecture 12* Equality of sine and the geometrically defined sine. Circular functions, hyperbolic functions and their inverses. Representation of the area-functions by logarithmic functions.	1.5
	Study of Lecture 12	4
To understand that differentiation is local approximation by a linear mapping. To be able to apply the rules for differentiation.	*Lecture 13* Different characterizations of the derivative. Rules for differentiation. Chain rule	1.5
	Study of Lecture 13	4
	Problems class 5	1.5
	Time for homework 5	10

Learning outcomes	Educational activities	Estimated student work time in hours
week 7		
To be able to deduce derivatives of inverse functions and to obtain inequalities by means of the mean value theorem.	*Lecture 14* The derivative of the inverse function. Mean value theorems Monotonicity and differentiation.	1.5
	Study of Lecture 14	4
Manipulating derivatives of power series within the radius of convergence.	*Lecture 15* Interchange of limit and derivative. Derivatives of functions defined by power series. Examples.	1.5
	Study of Lecture 15	4
	Problems class 6	1.5
	Time for homework 6	10
week 8		
To be able to apply Leibniz's rule and to produce the (formal) Taylor series of a given function.	*Lecture 16* The *n*th derivative of the elementary functions. Leibniz's Theorem. Theorem of Taylor.	
	Study of Lecture 16	4
To understand the ideas centred round Taylor's Theorem. To calculate power series of simple and more complicated functions.	*Lecture 17* Expansion of functions into power series. Application of Cauchy multiplication.	1.5
	Study of Lecture 17	4
	Problems class 7	1.5
	Time for homework 7	10
week 9		
To be able to investigate functions and to draw their graphs.	*Lecture 18* Convex and concave functions. Relative and absolute extrema. Points of inflection. Examples.	1.5
	Study of Lecture 18	4
To understand the concept of limits like lim $x \to \infty = -\infty$ To be able to apply the different versions of L'Hôpital's rule.	*Lecture 19* Extension of R to RU {-∞,∞}. L'Hôpital's Rule. Examples.	1.5
	Study of Lecture 19	4
	Problems class 8	1.5
	Time for homework 8	10

Learning outcomes	Educational activities	Estimated student work time in hours
week 10		
To be able to present the idea of the Riemann integral.	*Lecture 20* Survey on different integrals. Definition of Riemann integral. Riemann integrable functions.	1.5
	Study of Lecture 20	4
Calculation of infinite series by using results on Riemann integrals.	*Lecture 21* Darboux's Theorem. The vector space of R-integrable functions. Mean value theorems.	1.5
	Study of Lecture 21	4
	Problems class 9	1.5
	Time for homework 9	10
week 11		
To understand that integration is not simply ''anti-differentiation''. Knowledge of the antiderivatives of elementary functions.	*Lecture 22* Fundamental Theorem of Calculus. Change of variable theorem. Interchange of limit and integral.	1.5
	Study of Lecture 22	4
To learn manipulative skills of the technique of integration.	*Lecture 23* Various techniques of integration	1.5
	Study of Lecture 23	4
	Problems class 10	1.5
	Time for homework 10	10
week 12		
Calculation of more complicated integrals.	*Lecture 24* Advanced techniques of integration.	1.5
	Study of Lecture 24	1.5
Familiarity with tests of improper integrals of the first and second kind.	*Lecture 25* Improper integrals. Absolute and conditional convergence. Gamma- and Beta-function. Demonstration of the power of analysis.	1.5
	Study of Lecture 25	4
	Problems class	1.5
	Time for homework 11	10

Learning outcomes	Educational activities	Estimated student work time in hours
week 13		
To learn that analysis in more ''complicated '' spaces can be reduced to the analysis of known spaces.	*Lecture 26* Convergence in finite-dimensional Euclidean spaces. Independence of the norm under consideration.	1.5
	Study of Lecture 26	4
Calculations of limits of vector sequence and vector series, inner products, vector products, matrices and determinants.	*Lecture 27* Interchange of limits and multilinear maps. Examples.	1.5
	Study of Lecture 27	4
	Problems class 12	1.5
	Preparation for the final.	6
week 14		
To understand that topology is an axiomatic theory. To decide whether a set is open or/and closed(or not). To determine the interior and closure of given sets.	*Lecture 28* Topology in Euclidean spaces. Open and closed set. Interior and closure of a set. Accumulation points	1.5
	Study of Lecture 28	4
To decide whether a given set is compact or not.	*Lecture 29* Properties of open and closed sets. Compact sets. Theorem of Heine-Borel.	1.5
	Study of Lecture 29	4
	Problems class 12	1.5
	Preparation for the final.	6

Summary calculation of workload:

Week 1 has a workload of 12.5 hours.

Each week 2 - 12 has a workload of 22.5 hours (totally 247.5 hours).

Each of the last 2 weeks has a workload of 18.5 hours.

The sum is 297 hours. Together with the presence time of the test (= 3 hours) we have a workload of 300 hours.

Assessment:

Weekly homework is mandatory, the homework will be corrected each week, and it is necessary to have at least half of correct solutions to be admitted to the final test which lasts three hours. If the students are awarded at least half of the maximum points available in the final test, they receive 10 ECTS credits.

7.3. Monitoring

While the previous section provided an example of a planning form for a particular module, one should not underestimate the difficulties associated with measuring the time needed for the various learning outcomes: there is a need for regular monitoring and re-evaluation of the initial ECTS assignments. Non-traditional modes of learning, such as e-learning, present their own particular problems; thus a robust system of monitoring and re-evaluation should be able to deal with all learning modalities.

8. Learning outcomes and competences - level cycle descriptors

We note here that the Dublin descriptors and the Tuning generic competences are assumed here for Mathematics, as for all other subject areas, for the three cycles. We focus on the subject specific descriptors for Mathematics.

8.1. First cycle descriptors

On completion of a first cycle degree in Mathematics, students should be able to:

- show knowledge and understanding of basic concepts, principles, theories and results of mathematics;
- understand and explain the meaning of complex statements using mathematical notation and language;
- demonstrate skill in mathematical reasoning, manipulation and calculation;
- construct rigorous proofs;
- demonstrate proficiency in different methods of mathematical proof.

8.2. First cycle level descriptor

Level 1. *Content.* The Mathematics all scientists should know: basic algebra and arithmetic, linear algebra, calculus, basic differential equations, basic statistics and probability.

Skills. To complete level 1, students will be able to

(a) understand some theorems of Mathematics and their proofs;

(b) solve mathematical problems that, while not trivial, are similar to others previously known to the students;

(c) translate into mathematical terms simple problems stated in non- mathematical language, and take advantage of this translation to solve them.

Level 2. *Content.* Basic theory of the main "mathematical subjects", incorporating the subjects listed in section 6.3.1.2. Other mathematical subjects can also be included at this level.

Skills. To complete level 2, students will be able to

(a) provide proofs of mathematical results not identical to those known before but clearly related to them;

(b) translate into mathematical terms problems of moderate difficulty stated in non-mathematical language, and take advantage of this translation to solve them;

(c) solve problems in a variety of mathematical fields that require some originality;

(d) build mathematical models to describe and explain non-mathematical processes.

8.3. Second cycle descriptor

On completion of a second cycle degree in Mathematics, students should be able to:

— read and master a topic in the mathematical literature and demonstrate mastery in a reasoned written and/or verbal report;

— initiate research in a specialised field.

8.4. Third cycle descriptor

On completion of a third cycle (doctoral) degree in Mathematics, a candidate will have completed a significant piece of original research which is potentially publishable in an international mathematics journal.

9. Subject specific competences

It has already been noted that the three key skills that any mathematics graduate should acquire are:

— the ability to conceive a proof,

— the ability to model a situation,

— the ability to solve problems.

These are the key programme learning outcomes, reflected at different levels in the following list of *sample* subject specific competences.

9.1. First cycle

1. Profound knowledge of "elementary" mathematics (such as may be covered in secondary education).
2. Ability to construct and develop logical mathematical arguments with clear identification of assumptions and conclusions.
3. Capacity for quantitative thinking.
4. Ability to extract qualitative information from quantitative data.
5. Ability to formulate problems mathematically and in symbolic form so as to facilitate their analysis and solution.
6. Ability to design experimental and observational studies and analyse data resulting from them.
7. Ability to use computational tools as an aid to mathematical processes and for acquiring further information.
8. Knowledge of specific programming languages or software.
9. Capacity to work with mathematics in an interdisciplinary context.
10. Capacity to communicate mathematics to non-mathematicians.

9.2. Second cycle

1. Facility with abstraction including the logical development of formal theories and the relationships between them.
2. Ability to model mathematically a situation from the real world and to transfer mathematical expertise to non-mathematical contexts.
3. Readiness to address new problems from new areas.
4. Ability to comprehend problems and abstract their essentials.
5. Ability to formulate complex problems of optimisation and decision making and to interpret the solutions in the original contexts of the problems.
6. Ability to present mathematical arguments and the conclusions from them with clarity and accuracy and in forms that are suitable for the audiences being addressed, both orally and in writing.
7. Knowledge of the teaching and learning processes of mathematics.

Again it should be noted also that these competences are developed progressively throughout a programme: a mathematics programme does not contain any units specifically on proof construction, for example; rather, it is through practice in all course units that these skills are developed. Most of the competences defined for the first cycle could be included "at a higher level" on the second cycle.

9.3. Third cycle

Some competences will in turn be developed to a higher level in the doctoral cycle. Key competences at this cycle are

1. Ability to conduct individual significant original research.
2. Ability to present results of research to audiences of various levels
3. Ability to present the results of research in publishable form.

9.4. Commentary on some sample competences

Competence: Ability to formulate problems mathematically and in symbolic form so as to facilitate their analysis and solution.

This competence essentially involves the ability to express a simple problem in the form of an equation, to express a statement written in common language in symbolic/mathematical form and vice versa, and to be critical about the solution: to know when a solution is sensible. This can be developed through feedback on exercises, and through problem solving and project work, where the application can illustrate the reasonableness of the solution.

Competence: Ability to design experimental and observational studies and analyse data resulting from them.

One interpretation of this competence is that first cycle students should be able to design functioning code segments in a high level language, correct input errors (i.e., understand the mathematics of the syntax), and then interpret the data (for example a phase plane portrait). In general, as computer aided analysis becomes more and more common, ability to appropriately design experiments will become a skill of increasing importance. Lab sessions are the best environment in which to develop such skills.

Competence: Facility with abstraction including the logical development of formal theories and the relationships between them.

This includes the following "abilities":

— understanding what mathematical objects are,

— manipulating them under formal rules,

— distinguishing between correct and incorrect operations,

— understanding the role of axioms, definitions and theorems.

Students are introduced to a variety of formal mathematical theories. They explore the limits of the theories under study, and they learn how some aspects of reality can be transformed into a formal theory after excluding what is considered accidental for the particular problem. They study and understand some theorems, perform some manipulations under formal rules and check their work against the correct versions, which are supplied.

9.5. Generic competences

A successful programme in Mathematics, as with any other subject area, will also develop the valued generic competences which are analysed in depth in other Tuning documentation. Indeed some of the key subject specific competences can have generic analogues. Nonethless, for a Mathematics programme certain generic competences are particularly important, and among these are

- the ability to communicate at different levels and for different audiences;
- precise writing and oral expression;
- team work.

10. Commonly used approaches to Learning, Teaching and Assessment in Mathematics

The Mathematics Tuning group undertook an audit of, *inter alia*, modes of learning, teaching and assessment in the participating universities. The responses verify that teaching and learning takes place in combinations of the following:

- **Lectures.** These are seen as a very time-efficient way for students to learn part of the large material involved in the corpus of mathematics. Students would lose much valuable time if they were, independently, to assimilate this material independently from the literature. In some cases, students acquire prepared lecture notes or have a set textbook; in other cases the taking of notes is seen as part of the learning process. The ideal approach here may be subject dependent. Nonetheless, lectures continue to play a dominant role in mathematics teaching, not merely in terms of imparting course content, but also with respect to the development of the key competences, and this was reflected in the responses of the group members.
- **Exercise sessions**. These are organised most often in tandem with lectures. They occur as groups with supervision, or individually as homework with subsequent supervision of the results. The aim of the exercises is two-fold: understanding of the theoretical material through examples and applications to problems. These sessions are essential in mathematics, where understanding is acquired by practice, not memorisation. Competences such as teamwork and presentation skills may also be developed in such sessions.
- **Homework.** While demanding on the time of the lecturer and/or teaching assistant, homework is clearly one of the most effective ways in which students can be encouraged to explore the limits of their capabilities. Group members have recorded its use in remedying deficiencies in students' knowledge of "elementary mathematics", which ideally should have been already acquired at secondary school. In this case it functions as a diagnostic test of the students' (developing) skills, which are developed

in an additional voluntary course concerning these basic skills. Homework, of course, allows feedback to the students, which gives them a clearer picture of their performance; however, while homework is often assigned, it is less often graded, except where classes are small.

— **Computer laboratories.** These are perhaps the most significant change in the teaching of mathematics in recent years, introducing an experimental aspect to the subject. They feature not only in computer science related and computational courses, but also in statistics, financial mathematics, dynamical systems etc. Laboratory sessions are also seen as by far the best environment in which to develop the ability to use computational tools as an aid to mathematical processes and for acquiring further information, the importance of which can only increase as computer aided analysis becomes more common.

— **Projects**. These are done individually or in small groups, and typically involve putting together material from different subfields to solve more complicated problems. Small group projects can help to develop the ability to do teamwork (identified as an important transferable skill). The projects may involve significant computational elements, as in the case of the computational competences referred to above. Throughout, the emphasis should be on understanding the mathematics and its interpretation; thus on learning progressively to pass from a problem to its mathematical model, to the solution of the mathematical problem and finally to the interpretation of the solution in terms of the original problem. Projects, particularly significant final year projects where they exist, also afford the opportunity to develop students' verbal and written communications skills.

— **Search of bibliography**. Both in libraries and on the internet, familiarity with efficient ways to obtain relevant information must be acquired, in particular at second cycle level.

— **Dissertation**. In a second cycle or Masters programme, a substantial individual piece of work should be accomplished in the last year, as a final step towards independent practice of mathematics. It could take different forms depending on the sub-field, but would be characterised by its level and workload.

— **Mathematics Learning Centres.** As a mechanism to deal with some students' difficulties with the transition from second to third level mathematics, many universities now have Mathematics Learning Centres, which typically provide flexible, student-centred approaches to learning for these students. Diagnostic tests are also used to identify gaps in their knowledge of elementary mathematics. Such complementary learning could also possibly aid the transition from the first cycle to the second, particularly for students with different first cycle profiles

Other complementary modes of learning include

— **Internships**. Internships in industry or business can be useful for some specialisations.

— **Mathematics workshops or study groups**. These provide the opportunity for (typically) 2nd and 3rd cycle students to spend some time (usually 4 or 5 days) solving industrial or business problems in an interactive mode with senior colleagues. Well established examples of these are the European Study Groups with Industry[5] and Mathematics in Medicine Study Groups.[6]

— **E-learning.** Apart from its use in computer laboratories, information technology allows students the opportunity to engage in complementary self directed learning, to explore abstract concepts interactively and also provides an alternative method for feedback and assessment.

10.1. Feedback and assessment

Assessment is mostly by written or oral end-of-semester examination, often supplemented by midterm examinations, homework exercises, and where relevant project assignments and programming assignments. If end of semester examinations are the sole assessment there is of course less feedback, and therefore less opportunity to learn through assessment, available to the students. On the other hand, lack of funding often limits the availability of continuous assessment. It has been noted that shortcomings in students' understanding of what is required of them often only becomes apparent at the time of assessment.

5 http://www.maths-in-industry.org

6 http://www.maths-in-medicine.org

Final year projects and second cycle dissertations have feedback built in as part of the supervision process. Some students perform better in this situation than in the traditional examination format. They also afford the opportunity to assess the acquisition of the generic and subject specific competences for each cycle.

10.2. Conclusion

Different modes of teaching and learning have a part to play in a mathematics programme, with some more appropriate to particular sub-fields and particular competences. In many universities, limitation of study time and lack of funding have pushed the balance towards time and cost efficient methods, mostly lectures and tutorials, except perhaps for a dissertation in the final year. The implementation of the Bologna declaration should be the opportunity to introduce more student centred teaching modes, to supplement the traditional ones.

11. Quality Enhancement

Tuning sees its particular role as that of encouraging quality enhancement -. i.e., "a constant effort to improve quality of programme design, implementation and delivery" - at programme level and providing tools to develop it. The Tuning approach is based on a set of consistent features:

— an identified and agreed need;

— a well described profile;

— corresponding learning outcomes phrased in terms of competence;

— the correct allocation of ECTS credits to the units of the programme;

— appropriate approaches to teaching, learning and assessment.

This approach is examined more thoroughly in the general introduction to Tuning[7] and is as applicable to Mathematics as to any other subject area.

We hope that this short document may be of assistance to anyone who wishes to initiate or review a programme in Mathematics or with a significant mathematical component.

7 González, Julia and Robert Wagenaar, eds. *Tuning Educational Structures in Europe. Universities' Contribution to the Bologna Process. An Introduction.* (Bilbao and Groningen, 2006) 152 pp.

12. List of Mathematics Subject Area Group (SAG) members

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Note 1: For Austria the Technical University Graz was a member institution in Tuning I, II and III, represented by Günter KERN.

Note 2: For Italy and the University of Pisa Andrea MILANI was the representative in Tuning I and II.

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